28 Bridging the Gap between Cognition and Developmental Neuroscience: The Example of Number Representation

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ABSTRACT Developmental cognitive neuroscience necessarily begins with a characterization *of* the developing mind. One cannot discover the neural underpinnings of cognition without detailed understanding *of* the representational Capacities that underlie thought. Characterizing the developing mind involves specifying the evolutionarily given building blocks from *which* human conceptual abilities are constructed. describing what develops, and discovering the computational mechanisms that underlie the process of change. Here, 1 present the current state of the art with respect to one example of conceptual understanding: the representation of number. Lessons for developmental cognitive neuroscience are drawn at two levels of analysis: first. structural analogy between *de*velopmental processes described at the neural level and at the cognitive level. and second. challenges posed u the level *of* systems neuroscience.

Bridging biology and psychology

The past 20 years have witnessed stunning advances in our understanding of how the brain develops (for reviews aimed at drawing implications for cognitive neuroscience. see Gazzaniga. 1995, 1999; Johnson. 1997: Quartz and Sejnowski, 1997). Similarly, new techniques for experimental work with human infants have yielded a wealth of evidence concerning the unfolding of early perceptual, conceptual, and linguistic capacities (for illustrative studies, see Baillargeon. 1993; Carey and Spelke, 199-1: Kuhl, 1999; Leslie, 1994; Mandler. 2000; Marcus et al., 1999: Saffran, Aslin. and Newport, 1996; Spelke et al., 1992; Teller. 1999). 111 spite of the advances on each of these fronts. Rakic's comment in his introduction to the Neural and Psychological Development section of the 1995 version of *Die Cognitive Neurosciences* between neuroanatomy, neurochemistry n euro-development, on the one hand, and the . behavio ral r

(See Nelson and Bloom, 1997. for a similar lament.) What. is needed to bridge the gap, to forge less tenuous connections:

SEEKING STRUCTURAL PARALLELS; STRUCTURAL ANALOGY Developmental neurobiology aims to undo stand the mechanisms underlying cell differentiation n, cell formation, cell migration, axon migration. formation, dendrite growth, and so forth. Developmental psychology (of humans) aims to understand the for- of the functional architecture of die mind (how memory. emotion, attention, executive function, etc... come into being) as well as the origins of concepts, language, and the computational capacities that constitute human reason.

Biology and psychology provide different levels of analysis, but practitioners of the two disciplines share fundamental intellectual goals. Both seek to understand the role of the genetic code in structuring the developing brain and mind, and both seek to specify the huge variety of processes that determine the path to the adult state. Both assume that the functional architecture of the brain, its connectivity and patterns of neural activity, provide the underpinnings of the mind, and both assume that a functional account of what the brain does (i.e.., an understanding of the mind) is a necessary guide to our understanding of how the brain works. Therefore, detailed understanding of mechanism at one level of analysis is, at the very least, a source of structural analogy for mechanism at the other.

We appeal to *a structural analogy* whenever we see a process at one level as similar to a process at the other, in the absence of any claim that we have reduced one to the other or that we have actually discovered the neural underpinnings of some cognitive ability or some

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developmental change. For example, we make a structural analogy when we see the similarity between the processes of overproduction and subsequent selection in synaptic development (e.g.., Rakic et al., 1986; Rakic, Bourgeois, and Goldman-Rakic, 1994) and the processes of selection in phonological development (Kuhl et al., 1992; Werker and Lalonde, 1988) or parameter setting in syntactic development. In these examples, going beyond structural analogy would require discovering what processes of neural selection (if any) subserve losing access to previously available phonological distinctions or choosing among previously specified syntactic parameters. Similarly, we make a structural analogy when we see the similarity between neural activity-driven mechanisms of axon or dendrite growth and evidence-driven formation of new representational resources (Quartz and Sejnowski, 1997). Going beyond this structural analogy would require discovering actual cases of axon or dendritic growth that underlie any actual case of perceptual or cognitive development.

Structural analogies are useful, for they force us to think of mechanisms at a very abstract level. But we must not be fooled into thinking that they go very far in closing the gap. Going beyond structural analogy will require discovering the neural mechanisms underlying actual cases of representational change. Because structural analogies hold at an abstract level, they are fruitful as each discipline looks to the other for broad insights into how to think about development, such as how to separate the role of genetically specified constraints from the role of experientially derived information on development, the relative roles of selective and constructive processes in development, continuity versus discontinuity in development, and so on. For example, developmental psychologists who worry about whether sense can be made of the claim that representational and computational capacities, even knowledge itself, may be innate should study neurobiology. On the assumption that patterns of neural connectivity underlie representational and computational capacities, studies of genetic constraints on the growth of neural connectivity provide a relevant existence proof for genetic specification of such capacities, albeit only at the level of structural analogy. There is an explosion of knowledge concerning the genetic mechanisms subserving brain development. For instance, Levitt (1999) described the establishment of the earliest rough map of the nervous system-the hierarchy of molecular interactions that determine cell differentiation and the production of axon guidance proteins. Rakic (1999) described the mechanisms by which the total size of the cerebral cortex is determined, as well as ways in which the six cortical layers are formed. Similarly, much is known about the regionalization of the cerebral cortex, and the great variety of mechanisms by which its final cytoarchitectonic structure is determined.

Besides the intrinsic fascination of this elegant work, there are many lessons for developmental psychologists. First (and most obviously), stimulus-driven activity cannot modulate brain connectivity until there is enough connected to support the activity. Even the earliest patterns of connections are highly organized, and largely under genetic control. Second, this work shows that it is possible to disentangle genetic and (chemical) environmental factors in determining the precise course of neural development, and to further disentangle these two together from the role of both intrinsically and extrinsically driven neural activity. Progress in establishing the role of genes in determining the details of neural development lends hope that there can he comparable progress in establishing the respects in which genetic factors contribute to the development of cognitive architecture and conceptual representations of the world.

Conversely, behavioral work establishes early emerging representational capacities (providing evidence relevant to *possible* evidentiary sources of the relevant knowledge). For example, evidence that neonates prefer face to nonface stimuli (cf. Morton. Johnson, and Maurer, 1990) suggests innate face representations that must not have needed extended exposures to faces for their construction, and looking time experiments that show that 2-month-olds represent objects as spatiotemporally continuous, even under conditions of occlusion, and subject to solidity constraints (e.g., Spelke et al., 1992) shows that experience manipulating objects is not necessary for these developments. In addition, formal work (e.g., learnability proofs; see Gold, 1967. Osherson, Stob, and Weinstein, 1986, Wexler and Culicover, 1980: connectionist modeling; see Elman et al., 1996) explores how different models of the learning process, different ideas about the initial state, and different ideas about the final state constrain each other. Such work also contributes to conceptual clarity with regard to distinguishing nature from nurture and studying their interactions.

Still, one must not oversell what. can he learned from structural analogy. Structural analogy is important *because* of the gap between what is known about conceptual development from behavioral and computational work, on the one hand, and developmental neurobiology, on the other. It's because we don't yet know how *any* conceptual knowledge, language, or reasoning is represented in the nervous system (at the level of neural connections and activity) that we must appeal to structural analogy in our thinking about how the developing brain underlies and in turn is shaped by the developing mind.

CLOSING THE GAP-THE LEVEL OF SYSTEMS NEUROSCIENCE, DIAGNOSTIC MARKERS Besides seeking structural parallels between mechanisms of developmental change specified at different levels, there are many more direct methods of building bridges between the two disciplines, as the chapters in the present volume attest. The most obvious derives from a premise that articulates much of the discipline of cognitive neuroscience: Large regions of the brain participate in specialized neural circuits, the functions of which may be studied through a combination of research methods-behavioral, imaging, computational, and lesion (experimentally produced in animal studies, neuropsvchological studies with humans). One descriptive goal of developmental cognitive neuroscience is to discover when functions known to be subserved by specific brain regions come on-line. In some cases voting children fail tasks on which failure is a *diagnostic marker* of specific brain damage in adults. Such a finding is a source of hypothesis-perhaps the child's failure is due to immaturity of the neural structure required for adult performance. Of course, there are always alternative explanations for the young child's failure. It coati- be, for instance, that the child simply has not vet mastered the knowledge required to perform the task. Such alternatives may be empirically tested. Notice that such research only partially fills the gap-and that. only insofar as the neural substrate of the computations carried out by the circuit is full understood, as well as the neurobiological mechanisms underlying the development of the circuit. These conditions, I submit, have not yet been met in any domain of perceptual, conceptual, or linguistic representation.

Goldman-Rakic (Goldman, 1971) provided a paradigmatic example of diagnostic marker research. She noted that young rhesus monkeys (less than 2 months of age) performed as did adult monkeys with lesions in dorsolateral prefrontal cortex on the marker task of delaved response. Shown food hidden in one of two or more wells, distracted, and then allowed to search, adult monkeys with prefrontal lesions (and young infant monkeys) often fail to retrieve the food, especially with repeated trials and with delays of more than a few seconds. Rather, a variety of perseverative patterns of response emerge. The infant performance develops to adult levels by 4 months of age. Infant monkeys lesioned in dorsolateral prefrontal cortex do not undergo normal development; their performance never improves, bolstering the inference that normal development involves dorsolateral prefrontal maturation.

This particular case is of great interest to developmental psychologists because of the observation that the delayed response task is almost identical to Piaget's object permanence task (Piaget, 1955). Diamond and Goldman-Rakic (1989; see also Diamond, 1991) established that infant monkey development on the A/not B object permanence task between ages 2 and 4 months matches, in parametric detail, human infant development between 7 and 11 months, implicating frontal development in the human case as well. Confirmatory evidence is provided by development of other marker tasks for frontal development, such as detour reaching, that are conceptually unrelated to object permanence. Thus, this work provides an alternative explanation to Piaget's for young infants' failures on object permanence tasks. Rather than reflecting different representations of objects, as Piaget believed, the failures reflect immature executive function (means/ends planning, inhibition of competing responses, deployment of working memory).

We do not yet know the nature of the maturational change involving prefrontal dorsolateral cortex between 2 and 4 months in rhesus and between 7 and 11 months in humans, specified at the level of the neurobiological mechanisms. The method of diagnostic markers does not completely close the gap: it establishes phenomena related to developing brain function for which neurobiological underpinnings are vet to be worked out.

OTHER PARTIAL STEPS The method of diagnostic markers depends upon studies of brain-damaged human beings and lesion studies in animals. Studies of brain-damaged children (whether due to lesions or genetic abnormalities) add relevant data concerning dissociations of function and plasticity during development. Noninvasive imaging techniques add new methods to the arsenal of tools for bridging the disciplines at the level of systems neuroscience (for a discussion of how imaging techniques might serve this purpose, see Nelson and Bloom, 1997). These tools are by nature interdisciplinary; they require precise psychological task analysis to provide interpretable data concerning the functions of the brain regions implicated in any given lesion or imaging study.

A common thread runs through my remarks so far-the importance of understanding the mind to understanding the brain. Before turning to an extended developmental example, I explicitly defend this assumption.

Starting from an understanding of the mind

It is obvious to any psychologist, rightly or wrongly, that any functional/computational understanding of the brain must begin with a functional/computational understanding of the mind-of perception, action, language, cognition, and reasoning. Consider those cases in which satisfying work bridging the gap has been accomplished. Much is known, for example, about how neural computations subserve color vision, from retinal processes on (e.g., Cornsweet, 1970; Wald, 1950). But *how* the brain codes for color vision was understood, at least in broad outlines, for decades before anything about the neural implementation was discovered-from straightforward psychophysical studies of color mixture, color contrast, and from studies of abnormal color vision (color blindness). Research that stemmed from Helmholz (1910) and from Hering (1920) guided the search for neural underpinnings. How could it be otherwise, given the bewildering complexity of the brain?

Generalizing this point to the topic at hand, it is equally obvious that *developmental cognitive neuroscience* must begin with a functional/computational understanding of the developing mind. This requires empirical study of the initial representations of the world available to infants, including a specification of their representational format, and detailing the computations performed over them. It also requires characterizing *development,* specifying the ways in which these initial representations become enriched over time and specifying the learning and maturational mechanisms involved.

The study of the development of numerical representations bears on two fundamental debates couched at the level of structural analogy. One concerns the relative importance of selection mechanisms and constructive mechanisms in both neural development and cognitive development (for a summary of this controversy and a defense of the importance of constructivism, see Quartz and Sejnowski, 1997). The two parallel selection/constructivism debates (within psychology and within neurobiology) are merely structural analogies. For example, at present there is no way to bridge the gap between parameter-setting models of language acquisition, or the narrowing of phonological representations (selection processes), and mechanisms of pruning of synaptic connections (also a selection process). These are *far* analogies; it is quite possible that the mechanisms that underlie parameter setting or the narrowing of phonological space are not neural selection processes of the sort that prune synaptic connections. At the level of cognitive development, a related debate concerns continuity of representational resources. Does cognitive development involve, at least in part, the construction of genuinely new representational resources, or is the strong continuity hypothesis of theorists such as Fodor (1975), Macnamara (1982, 1987), and Pinker (1984) true? If constructivism is correct, through what mechanisms are new representational resources created? These questions must be answered at psychological and computational levels before any search for neurological underpinnings may even begin.

In the rest of this chapter, I shall argue on the developmental discontinuity and constructivist side of the debates, at least in the case of number representations. The point of this example is to show what sort of understanding of the developing mind is necessary to begin to bridge the gap. I shall argue that the level of detail concerning representational format that is need to inform current debates at the level of structural analogy is also needed to guide the search for neural underpinnings at the level of systems neuroscience.

An extended example-The development of number representations

PRELINcuISTrc Representations ofi
from many different paradigms provide convergent evidence that preverbal infants represent individual tokens of various types (events, syllables. objects) and keep track of the number of individuals in an array, in at least some contexts. Habituation experiments provide one sort of evidence. In these, babies are presented with arrays containing a fixed number of individuals until they decrease their attention to the arrays, and their sensitivity to numerosity is inferred from their recovery of interest to new arrays containing a new, but not the old, number of individuals. Successful discrimination of 1 vs. 2 or 2 vs. 3 individuals has been shown for syllables (Bijeljac-Babic, Bertoncini, and Mehler, 1991); for successive actions (Wynn, 1996); for dots varying in position (Antell and Keating, 1983, Starkey and Cooper, 1980), for pictures of objects varying in shape, size, and position (Strauss and Curtis, 1981; Starkey, Spelke, and Gelman, 1983); and for moving forms undergoing occlusion (Van Loosbroek and Smitsman, 1990), at ages varying from birth to 6 months. In contrast, infants often fail to discriminate 3 from 4 elements (Strauss and Curtis, 1981) and they consistently fail to discriminate 4 from 5, 4 from 6, or 8 from 12 elements (Strauss and Curtis, 1981; Starkey and Cooper, 1980; Xu and Spelke, 2000). In one study, however, 6-month-old infants were found to discriminate 8 from 16 dots (Xu and Spelke, 2000). As we shall see, not all of these

studies adequately controlled for *nonnumerical* bases of discrimination, but others (e.g., Wynn, 1996; Xu and Spelke, 2000) did. Insofar as the controls are adequate, these studies reveal representation of number in the sense of discrimination on the basis of numerical differences. But confidence in these representations as *number* representations requires more; it is important that computations defined over them have at least some of the conceptual role of number representations. This condition is met: Infants use their representations of number to compute more/less and carry out simple addition and subtraction.

Some of the evidence for number-relevant computations derives from experiments using the violation of expectancy looking time method. Wynn's (1992a) classic addition/subtraction studies, now replicated in many laboratories (Koechlin, Dehaene, and Mehler, 1997; Simon, Hespos, and Rochat, 1995; Uller et al., 1999) suggest that babies represent the numerical relations among small sets of objects. In a typical experiment, an object is placed on an empty stage while the baby watches; then a screen is raised that covers the object, a second object is introduced behind the screen, anti the screen is lowered to reveal either one object or two. In this 1 + 1 = 2 vs. I event, infants of ages ranging from 4 to 10 months look longer at the impossible outcome of 1 than at the possible outcome of 2. That infants expect precisely 2 objects is shown *by* the fact that they also look longer at the impossible outcome of 3 in a I + 1 = 2 vs. 3 event (Wynn, 1992a). Infants also succeed at 2 - 1 = 2 vs. 1. Success with larger numbers such as 3 - 1 and 2 + 1 is less robust (Baillargeon, Miller, and Constantino, 1993; Uller and Leslie, 2000).

Preliminary data from a third paradigm, developed by Hauser, Carey, and Hauser (2000) to study spontaneous representation of number by free-ranging rhesus macaques, suggests that prelinguistic infants represent the ordinal relations among small sets of different numbers of objects. Each infant gets only one trial; these are totally spontaneous number representations. Ten- and 12-month-old infants watched as an experimenter put a number of small cookies successively into one opaque container and a different number into a different opaque container, whereupon they were allowed to crawl to the container of their choice. Infants of both ages approached a container with 2 cookies over a container with just I cookie, and a container of 3 cookies over one with just 2 cookies. Infants of both ages failed at choices involving 3 vs. 4, as well as at choices involving 3 vs. 6; see figure 28.1 (Feigenson et al., 2000).

As in Wvnn's paradigm, infants must compute 1 + 1 or 1 + I + l. In addition, this is the first demonstration of ordinal comparison of two sets represented in memory in prelinguistic infants. In sum, the violation of expectancy studies and the choice studies suggest that preverbal infants discriminate among sets on the basis of numerosity and represent numerical relations among sets, such as 1 + 1 = 2 and 2 > 1. Numerical representations predate language learning, and thus the mechanisms by which they are formed do not involve mastering the culturally constructed, explicit, integer-list representations of natural language. Indeed, these demonstrations open the question of continuity. Is the integer-list representation ("one, two, three, four, five, ...") and the counting algorithm merely an explicit form of the representational system underlying these infant capacities? Or is the integer-list representation discontinuous with those that underlie the infant's behavior? To begin to answer this question, we must consider the format of infants' representation of number.

GELMAN AND GALLISTEL's CONTINUITY HYPOTHESIS Gelman and Gallistel (1978) suggested that infants establish numerical representations through a nonverbal counting procedure: Babies represent a list of symbols, or "numerous," such as &, ^, #, $, ©. Entities to be counted are put in 1:1 correspondence with items on this list, always proceeding through it in the same order. The number of items in the set being counted is represented by the last item on the list reached, and its numerical value is determined by the ordinal position of that item in the list. For example, in the list &, ^, #, $, ©, the symbol ^ represents 2, because ^ is the second item in the list.

Gelman and Gallistel's proposal for the nonlinguistic representation of number is a paradigm example of a continuity hypothesis, for this is exactly how languages with explicit integer lists represent the positive integers. On their hypothesis, the child learning "one, two, three, four, five, ..." need only solve a mapping problem: identify the list in their language that expresses the antecedently available numeron list. Originally learning to count in Russian should be no more difficult than learning to count in Russian once one knows how to count in English.

WYNN's EASE OF LEARNING ARGUMENT AGAINST THE GELMAN/GALLISTEL CONTINUITY PROPOSAL Between the ages of 2 and 4 years, most children learn to count; and contrary to the predictions of a numeron-list theory, learning to count is not a trivial matter (Fuson, 1988; Wynn, 1990, 1992b). Of course, this fact in itself does not defeat the continuity hypothesis, for we do not know in advance how difficult it is to learn an arbitrary list of words or to discover that one such list (e.g., "one,

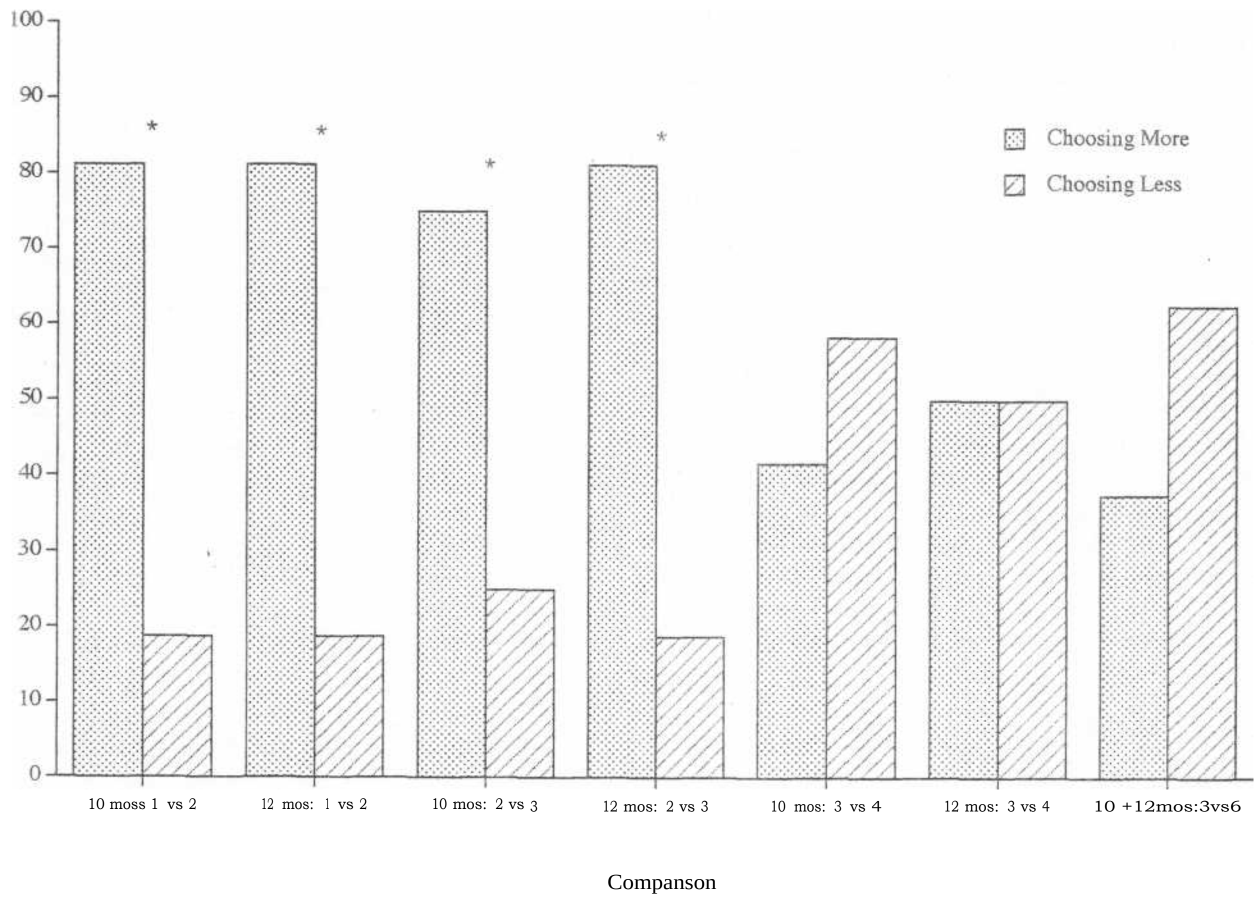

FIGURE 28.1 Percent choice of container with greater number of crackers.

two, three, ..." rather than "a, b, c, ..." or "Monday, Tuesday, Wednesday,) is the list in English that represents number. However, Wynn's (1990, 1992b) studies show that children have difficulty discovering the meanings of specific number words even after they have solved these two problems. The sequence in which children learn the meanings of the number words therefore is at odds with that predicted by the continuity thesis.

Wynn began by confirming that young children do not know the numerical meanings of the words in the count sequence. First, she identified children who could count at least to six when asked "how many" objects there were in an array of toys. These 2-3-year-old children honored 1:1 correspondence in their counts, and used a consistently ordered list, although sometimes a nonstandard one such as "one, two, four, six, seven, ..." She then showed that if such a child were given a pile of objects and asked to give the adult "two" or "three" or any other number the child could use in the game of counting, most 2- and 3-year-old children failed. Instead, young children grabbed a random number of objects (always more than one) and handed them to the experimenter. Also, shown two cards depicting, for example, 2 vs. 3 balloons and asked to indicate which card had 2 balloons on it, young children responded at chance. Analyses of within-child consistency between the "give *n*" and "which card has *n*" tasks bolstered the conclusion that young children count for more than a year before they learn what the words in the count sequence mean: i.e., before they learn how "one, two, three, four, five, ..." represents number.

There is one more observation of Wynn's that is important to the evaluation of the Gelman/Gallistel continuity hypothesis. Wynn (1990, 1992b) showed that from the beginning of learning to count, children know what "one" means. They can pick *one* object from a pile when asked, and they correctly distinguish a card with "one fish" from a card with "three fish." Furthermore, they know that the other words in the count sequence contrast with one. They always grab a random number of objects greater than one when asked to hand over "two, three, four, ..." objects, and they also successfully point to a card with three fish when it is contrasted with a card with one, even though their choices are random when "three" is contrasted with "two." Thus, Wynn's studies provide evidence that toddlers learn the English count list and identify the list as relevant to number very early on (younger than age 2V2) : They know what "one" means, they use "two, three, four, etc.," in contrast with one," and they only use the number words above one when presented with sets greater than one. They are in this state of knowledge for a full year before they work out the principle that allows them to determine which number each numeral refers to. This state of affairs is impossible on the numeron-list continuity hypothesis, whereby the English count list need only be identified and mapped onto the preexisting nonlinguistic numeron list that the infant already uses to represent number.

Wynn's work is important for another reason, because her data constrain our hypotheses about the process through which children construct the integer-list representation. Her studies show that children learn the meanings of the number words as follows: First, they learn what "one" means, as indicated above, with other number words contrasting with "one": then they learn what "two" means. with other words contrasting with "two" and referring to numbers bigger than 2. Wynn's longitudinal data established that some children are in this state for several months. Then they learn what "three" means; some children induce the meaning of all the number words in the list at this point; others know what "three" means, and take the other words to contrast with "three" and refer indiscriminately to higher numbers. Wvnn found no children who knew what "`foul" meant who had not worked out how the whole list represents number.

In spite of the evidence that prelinguistic infants represent number, it seems that the child is still in the process of constructing an integer-list representation of number in the years 2-4. How then do infants represent number, and how do their representations differ from the integer-list representation

Two POSSIBLE PRELINGUISTIC REPRESENTATIONAL SYSTEMS FOR NUMBER Two quite different nonlinguistic systems of number representation have been proposed to underlie infants' abilities to discriminate, compare, and compute over representations of number: analog-magnitude representations and object-file representations. I characterize each in turn, considering the evidence for and against the hypothesis that each one may underlie the behaviors in which infants display knowledge of number.

-lnalog-magnitude representations Both human adults and animals deploy analog-magnitude representations of number (for reviews, see Gallistel, 1990; Dehaene, 1997). Rather than being represented by a list of discrete symbols, in such systems number is represented by a physical magnitude that is proportional to the number of individuals in the set being enumerated. An external analog-magnitude representational system could represent one as --, two as -- , three as ----, and so on. In such systems, numerical comparisons are made by processes that operate over these analog magnitudes, in the same way that length or time comparisons are made by processes that operate over representations of these physical magnitudes. For this reason, number comparisons are subject to Weber's law (1 and 2 are more discriminable than are 7 and 8). Sensitivity to Weber's law is one of the main sources of evidence that both animals and human adults prevented from verbal counting deploy analog-magnitude representations of number (for reviews of other sources of evidence, see Dehaene, 1997; Gallistel, 1990).

There are many different wavs that analog-magnitude representations of number might be constructed. One proposal is the accumulator model of Neck and Church (1983). The idea is simple: Suppose the nervous system has the equivalent of a pulse generator that generates activity at a constant rate, and a gate that can open to allow energy through to an accumulator that registers how much has been let through. When the animal is in a counting mode, the gate is opened for a fixed amount of time (say 200 ins) for each individual to be counted. The total energy accumulated then serves as an analog representation of number. Meck and Church's model seems best suited for sequentially presented individuals, such as bar presses, tones, light flashes, or jumps of a puppet. Gallistel (1990) proposed, however, that this mechanism functions as well in the sequential enumeration of simultaneously present individuals. Others (e.g., Dehaene and Changeux, 1989; Church and Broadbent, 1990) have proposed computational models that create analog-magnitude representations without implementing any iterative process. Furthermore, there is unequivocal evidence that in at least some circumstances, number representations are created by noniterative processes. For example, the time that subjects require to discriminate two numerosities depends on the ratio difference between the numerosities, not on their absolute value (Barth, Kanwisher, and Spelke, under review). In contrast, time should increase monotonically with N for any iterative counting process. Moreover, subjects are able to discriminate visually presented numerosities under conditions of stimulus size and eccentricity in which they are not able to attend to individual elements in sequence (Intrilligator, 1997). Their numerosity discrimination therefore could not depend on a process of counting each entity in turn, even very rapidly.

Analog-magnitude representations are numerical in four senses. First, and most importantly, each analog magnitude stands for a numerical value, albeit imprecisely. Second, the processes for constructing such representations are subject to no upper bound beyond those imposed by perceptual thresholds such as the discriminability of interelement distances (e.g., individual tones or dots may fail to be resolved if they are too crowded) and the ultimate capacity of the system. Third, distinct analog magnitudes provide a natural and explicit representation of more/less relationships between distinct numerosities, with the larger magnitude representing the more numerous set. Fourth, analog magnitudes provide a medium for computations of addition and multiplication, at least in principle (see Gallistel and Gelman, 1992). although it is an empirical question whether they are used for this purpose in practice. Studies of neurological patients with dvscalculia and of bilingual students performing arithmetic suggest that analog magnitudes are used to estimate sums but not products, at least in these populations (Dehaene, 1997; Dehaene et al., 1999; Spelke and Tsivkin, 19.99). Studies of patients with dyscalculia also suggest that analog-magnitude representations are recruited during single digit subtraction and in computations of "between" (what is between 6 and 8, 3 and 5, etc.; Dehaene, 1997).

Analogy magnitude representations differ from other numerical representations (in particular, from any representation of the natural numbers, including integer-list representations) in some crucial respects. Because such representations are inexact and subject to Weber fraction considerations, they fail to capture small numerical differences between large sets of objects. The distinction between seven and eight, for example, cannot be captured by the analog-magnitude representations found in adults. Also, noniterative processes for constructing analog-magnitude representations, such as those proposed by Dehaene and Changeux (1989) and Church and Broadbent (1990), include nothing that corresponds to the successor function, the operation of "adding one," and they do not capture the fact that 1 and 2, 7 and 8, and 69 and 70 differ by the same amount. Analog-magnitude representations therefore are not powerful enough to represent the natural numbers and their key property of discrete infinity, and they do not support any computations of addition or multiplication that build on the successor function.

Can analog-magnitude representations underlie human infants' sensitivity to number Such representations could underlie infants' discrimination of 2 individuals from 3 individuals, including syllables and jumps of a puppet. In analog-magnitude representations, these small sets would differ as ---- differs from ----. In contrast, noniterative analog magnitude processes could not easily represent the numbers in Wynn's addition and subtraction tasks, because they do not implement the successor function. Similarly, the ordinal comparison tasks in which one food object is placed into one closed container and two are placed into another, one at a time (or two vs. three, each set being constructed by successive placings), require that the infant represent "one" and succession. Because they lack any representation of "adding one," noniterative analog-magnitude representations provide a cumbersome procedure for representing the addition event itself in these studies: Infants would have to represent occluded as well as visible objects and perform density and extent measurements on the set of all the objects in an array, visible and occluded.

Further results provide other difficulties for this class of models. Analog-magnitude representations cannot account for the finding that infants discriminate 2 from 3 dots but not 4 from 6 dots (Starkev and Cooper, 1980) or 8 from 12 dots (XU and Spelke, 2000). Because these pairs of numerosities differ by a constant ratio, they should be equally discriminable by any process whose sensitivity follows Weber's law. Similarly, analog-magnitude representations also cannot account for the finding that infants succeed at addition tasks such as I + I = 2 or 1, but fail at 5 + 5 = 10 or 5 (Chiang and Wvnn, 2000). Nor can analog-magnitude representations account for the finding in the choice experiments (see figure 28.1) that infants can compare 1 vs. 2 and 2 vs. 3, but not 3 vs. 6. Infants' performance in these cases, as well, departs from Weber's law.

One set of findings, however, strongly supports the thesis that infants construct analog-magnitude representations of number. Xu and Spelke (2000) found that, when overall surface area, brightness, and density of dots was controlled across presentations, infants could discriminate 8 from 16 dots and failed to discriminate 8 from 12 dots under the same presentation conditions. Presented with simultaneously visible arrays of 4 or more individuals, infants' number representations appear to accord with Weber's law. Moreover, details of Xu and Spelke's (2000) data suggest that a noniterative process supports performance in this task. Infants took no longer to habituate to 16-dot arrays than to 8-dot arrays, suggesting that they formed representations of the different numerosities with equivalent speed, contrary to the predictions from any iterative process.

Thus, I conclude that infants *do* have available analog-magnitude representations of number, but that they are unlikely to underlie performance on many of the tasks involving small set sizes.

Object file representations A wealth of studies provide evidence that human adults can attend to small numbers of objects simultaneously, keeping track of each object by following it as it moves continuously and accumulating information about its properties (Kahneman, Treisman, and Gibbs, 1992; Pylyshyn and Storm, 1988). Adults are able to perform this task even when distinct objects have no distinguishing properties other than their location, and when a single object maintains no common properties over time as it moves. Conversely, when two objects with distinct properties share a common location (for example, a small circle is embedded in a large one), or when a single object moves discontinuously, adults sometimes fail to represent the two objects' distinctness or the single object's identity (Scholl and Pvlvshvn, 1999; Trick and Pylyshyn, 1994). A11 these phenomena testify to the existence of a spatiotemporal system for representing objects-up to about four of them at once-in visual scenes. Such object representations have been proposed to underlie adults' rapid apprehension of numerosity in arrays of simultaneously visible elements, when each element occupies a distinct location and the total number of elements does not exceed about 4 (Trick and Pvlvshvn. 1994). Such representations also have been proposed to underlie adults' rapid apprehension of the identity of individual visible object; through time and their accumulation of information about the properties of each object. These representations have been called by different names, and different theorists do not exactly agree as to their nataure or properties. Here, I abstract over these differences and, following Kahneman and colleagues (1992), refer to such representations as "object files."

Object-file representations are numerical in four senses. First, the opening of new object files requires principles of individuation and numerical identity; models must keep track of whether this object, seen now, is the same *one* as that object seen before. Spatiotemporal information must be recruited for this purpose, because the objects in many experiments are physically indistinguishable from each other and because, in any case, property/kind changes within an object are not sufficient to cause the opening of a new object file (for reviews, see Carey and Xu, in press; Pylyshyn, in press). Second, the opening of a new object file in the presence of other active files provides a natural representation for the process of adding one to an array of objects. Third, object-file representations provide implicit representations of sets of objects; the object files that are active at any given time as a perceiver explores an array determines a set of attended objects. If the perceiver can establish a 1:1 correspondence between objects and active files, therefore, the computations over object-file representations provide a process for establishing numerical equivalence. Fourth. object files represent numerosity exactly for set sizes up to about 4 and are not subject to Weber's law.

Object-file representations differ from integer-list representation in three ways. Most importantly, they contain no symbols for cardinal values. The only symbols in such models represent the individual objects themselves. Second, object-file models have an upper bound (of about 4; see Pvlvshvn and Storm, 1988; Trick and Pylyshvn, 1994). Third, object-file representations serve only to represent individuals that obey the spatiotemporal constraints on objects; they do not serve to enumerate events or other entities. Of course, the existence of object-file representations does not preclude similar representational systems for other types of individuals (event-file representations. syllable-file representations).

Many have suggested that infants construct mental models of the objects in number discrimination aancl addition experiments, deploying the object tracking capacities studied in the adult object-based attention literature to update these representations as objects are added and subtracted from the array. JCarev and Xu, in press: Leslie and Scholl. 1999; Simon, 1997: l'11cr et al., 1999). That is, infants may be constructing a representation consisting of one object file for each object in a given array. Object-file representations could underlie infants' discrimination of two individuals from three individuals; in these representations, the small sets of objects differ as 0 0 differs from 0 0 0. Object-file representations also provide a natural account for performance in the infant addition experiments. During the setup of a 1 + 1 = 2 or I experiment, for example, the infant would open an object file for the first object introduced on the stage and a second object file for the second object introduced behind the screen, yielding the representation 0 0. Then when the outcome array is revealed, the infant again creates object file representations, 0 0 for the expected outcome of two objects and 0 for the impossible outcome of 1 object. The model of the setup array and the model of the outcome array are compared on the basis of 1:1 correspondence, and a mismatch draws the infant's attention. Evidence in favor of this account comes from the finding that infants do not use property, ' kind information (e.g., the information that distinguishes a cat from a dog) to open new object files in addition experiments (Simon, Hespos, and Rochat, 1995; Xu and

Carey, 1996), just as adults fail to use such information in object tracking experiments (Pylyshyn, in press).

The pattern of success in the choice experiment (see figure 28.1) is the set-size signature of object-file representations. That the limit of performance in the choice experiments is 2 vs. 3, in the face of failure at 3 vs. 6, suggests that object-file representations underlie performance on this task. If so, these studies show that the child can create *two* models in short-term memory, each with a limit of 2 or 3, and compare them on the basis of 1:1 correspondence or overall spatial extent. Such representations also can explain limits in Wynn's addition/subtraction paradigm, such as their evident failure to recognize that 5 + 5 = 10 not 5.

Finally, compelling evidence in favor of object-file representations comes from studies comparing infants' ability to add objects to their ability to add other perceptible entities that violate the conditions on objects: piles of sand (Huntlev-Fenner, Caret'. and Solimando, under review) and piles of blocks (Chiang and Wvnn, 2000). For example, Huntley-Fenner and colleagues presented 8-month-old infants with a 1+ 1= 2 vs. I addition problem either with two solid objects of the shape, color, and texture of sand piles or with two real sand piles. Although these displays were closely matched on perceptual grounds, babies succeeded at the addition task with the solid objects and failed with the sand piles. This pattern strongly suggests that a process for individuating and tracking objects is central to performance in such addition experiments.

In further support of the hypothesis that object-file representations underlie performance on the infant addition/subtraction tasks, Uller and colleagues (1999) showed that manipulations that would affect the robustness of object-file representations held in short-term memory, but that should not affect analog-magnitude representations, have a large influence on infant success in these experiments. For example, suppose a symbolic representation of the number of a set of 2 objects is created by incrementing an accumulator twice. It should not matter whether the infant actually sees the first object on the stage floor before a screen is raised and the second object is added to the array (object first) or whether the screen is raised first and then the objects are added one at a time (screen first). However, if the child is creating and storing a mental model of the objects in the event, seeing the first object and requiring only one update in imagery might well have a huge effect. The latter result is obtained: Infants succeed in object-first 1 + I = 2 or 1 experiments by 4-5 months of age, but not in screen-first 1 + 1 = 2 or 1 experiments until 10 months of age (Uller et al., 1999).

Finally, recent findings from Clearfield and Mix (1999) and from Feigenson and colleagues (in press) strongly suggest that analog-magnitude representations *of number* do not underlie the habituation findings with small sets. These authors point out that no previous study of small number discrimination successfully controlled for the possibility that a continuous quantity correlated with number-such as the total spatial extent of the array-might be driving infants' responses in these studies. Attempted controls (e.g., Starkey and Cooper, 1980) failed-even when stimulus size is randomly varied across trials, the average spatial extent of the habituation series will be a closer match to that of the familiar number test items than to that of the novel number test items. When Clearfield and Mix (1999) and Feigenson and colleagues (in press) corrected this design flaw, no evidence for sensitivity to numerical differences between sets below 4 was found in 6- and 7-month-old infants. Rather, infants were sensitive to changes in total spatial extent of the arrays (total contour length in Clearfield and Mix; total front surface area in Feigenson et al.). Although the evidence from other paradigms that infants represent number stands, these results provide no evidence that infants represent number at all for very small sets when tested with a numerosity discrimination paradigm. In contrast, one experiment using the same type of discrimination paradigm and including proper controls for spatial extent and other continuous variables provides evidence for discrimination of numerosity ;Nu arid Spelke, 2000). It is noteworthy that in this study infant discrimination cannot be based on object files, for the numbers discriminated (8 vs. 16) are too large. The findings of Clearfield and Mix and of Feigenson and colleagues, however, are consistent with object-file representations, in which models of objects are compared with respect to the total spatial extent of the arrays.

THE FORMAT OF INFANT REPRESENTATION OF NUMBER

As the preceding discussion indicates, the issue *of* which nonlinguistic system of representation underlies performance in the infant number experiments is currently a matter of debate, and no single system of representation appears adequate to account for all the findings. Object-file representations appear to underlie performance in some tasks, but not all of them. It seems likely that prelinguistic infants, like nonhuman primates and human adults, have access both to object-file representations and to analog-magnitude representations. When infants are presented with small numbers of material objects in addition and subtraction experiments, they appear to keep track of those objects by means of object files. When they are presented with large sets of objects in discrimination experiments, they appear to represent those sets with analog-magnitude representations. When they are presented with small sets of objects in discrimination experiments, they may not represent numerosity at all. Finally, it is not clear whether infants keep track of small numbers of events by means of analog-magnitude representations or by an entirely different system of parallel individuation. The hypothesis that human infants are endowed both with analog magnitude and object-file representations of number is not implausible, because a wide variety of species, including rats, parrots, chickens, pigeons, and primates, command both systems of representations (for reviews, see Gallistel, 1990; Dehaene, 1997).

WHY THE INTEGER-LIST REPRESENTATION IS A GENUINELY NEW REPRESENTATIONAL RESOURCE The integer-list representation differs deeply from both object-file and analog-magnitude representations of number, and it transcends each of them singly and together. Object-file representations are sharply limited to small sets by constraints on parallel individuation and short-term memory for distinct items, whereas integer-list representations contain no such limit. There is no intrinsic upper bound on the lists that one can commit to long-term memory (think of the alphabet). Once an integer list embodies a recursive component for generating symbols in the list, there is no upper limit at all (think of the base-10 system). Moreover, object-file representations contain no symbol for any integer value and no counting algorithm, even implicitly, whereas integer-list representations are constituted by lists of explicit symbols for numerosities. Nonetheless, object-file representations have two important properties in common with integer-list representations. First, principles of individuation and numerical identity, plus the capacity to compare models on the basis of 1:1 correspondence, enable sets of 1, 2, and 3 individuals (1 + 1 = 2, 2 > 1, etc.) to be *precisely,* if *implicitly,* represented. Second, object files provide a natural representation of the successor function, for *adding one* corresponds to the opening of a new object file. In integer-list representations, the successor relation (+1) between adjacent numbers similarly enables the numerical relations among sets of individuals to be *precisely* encoded.

Analog-magnitude representations, like integer-list representations, have no intrinsic upper limit and contain a distinct symbol for each integer value represented (e.g., --- represents 2). Discriminability of these symbols, however, is limited by Weber's law, such that one cannot represent the precise relation between 14 and 15, although one may be able to represent that 15 is greater than 12. Integer-list representations, in contrast, capture all relations among whole numbers, no matter how close (14, 15, 114, 115, 116, etc.). Moreover, analog-magnitude representations, especially those produced by a noniterative process, obscure the arithmetic successor relation (+ 1) between successive states in two ways. First, there is no operation within the process that constructs these representations that corresponds to the operation of *adding one.* Second, Weber fraction considerations make the difference between the states that represent I and 2 appear larger than the difference between 2 and 3, or 29 and 30.

Even if the infant is endowed with both analog magnitude and object file systems of representation, therefore, the infant's capacity to represent number will be markedly weaker than that of the child who commands the integer-list representation. Neither object files nor analog magnitudes can serve to represent large exact numerosities: Object files fail to capture number concepts such as "seven" because they exceed the capacity limit of four, and analog-magnitude representations fail to capture such concepts because they exceed the limits on their precision. Children endowed with both these representations therefore lack the representational resources provided by the integer-list system.

In sum, a consideration of the format of infant number representation reinforces the conclusions from Wynn's ease of learning argument. The integer-list representation is discontinuous with its developmental precursors; acquiring it requires the construction of a new representational resource. How do children build an explicit, integer-list, representation of the natural numbers?

Constructing the integer-fist representation of number

The problem of how the child builds an integer-list representation decomposes into the related subproblems of learning the ordered list itself ("one, two, three, four, five, six, ..."), learning the meaning of each symbol on the list (e.g., "three" means *three),* and learning *how* the list itself represents number such that the child can infer the meaning of a newly mastered integer symbol (e.g., "eleven") from its position in the integer list. On Gelman and Gallistel's (1978) continuity proposal, the second and third questions collapse into the first-the child identifies a natural language list such as "one, two three four, five six, ..." as an integer list that corresponds to the mental numeron list, and in doing so knows the meanings of all its symbols at once. Empirical considerations against this proposal were presented earlier.

BOOTSTRAPPING MECHANISMS Presenting a full answer to the question of the mechanisms underlying the construction of the integer-list representation (a new representational resource) is beyond the scope of this chapter. Carey (in preparation) and Carey and Spclke (in press) provide an account of bootstrapping mechanisms in general, and sketch four different bootstrapping processes that might underlie this achievement. Bootstrapping processes have several properties. They often involve integrating previously distinct representational systems through processes that are not merely combinatorial; they are nonalgorithmic, involving such optional operations as creating analogical mappings between different representations and making inductive leaps. And they involve, initial stages of learning in which concepts are acquired directly in relation to each other before being grounded in the meanings they will initially attain, a process quite unlike abstraction. The four bootstrapping processes suggested by Carev and Spelke (in press) differ in the representations the child starts with, and thus in the analogies drawn and particular inductive leaps made. All four proposals assume, following WVynn (1990) and Fuson (1988), that the child first learns "one, two, three, four, five, ..." as a list of meaningless lexical items. There is no doubt that children have this capacity-they learn sequences such as ¨eenv, meenv. miney, mo," the alphabet, the (lays of the week, and so on. As Terrace (1984) has shown, nonhuman primates (rhesus macaques) also have the capacity to learn to order arbitrary lists of visual symbols; they can be taught to touch 4 pictures (e.g., clog, house, tree, cup), displayed in successive trials in randomly determined positions on a video screen, in a fixed order. (For a fascinating application of this ability in monkeys to the exploration of monkey number representation, see Brannon and Terrace, 1998). This step is a paradigmatic example of one aspect of bootstrapping processes: The meanings of the counting words are exhausted, initially, by their interrelations-their relative order in the list.

All four proposals also capture the fact that the meanings of small number words are learned first. The proposals differ in the learning processes proposed to underlie working out the meaning of these number words, of the larger number words, and of the counting algorithm. All agree that the learning process involves combining antecedently available representations. Bootstrapping learning processes begin in representations already grounded, as well as in representations that derive meaning from their interrelations. The proposals differ in which numerical system(s) available to infants-the analog magnitude system and the object-file system-play a role in the learning process, as well as to what additional representational resources are recruited.

A SKETCH OF ONE PROPOSAL Many authors (e.g., Hurford, 1987) have offered versions of the proposal that the integer-list system is built from object-file representations and the meaningless ordered list of words. For reasons too complex for present purposes, Spelke and I do not favor this proposal; but here I offer it as an illustration of how bootstrapping processes might create new representational resources.

First, infants spontaneously represent small numbers of objects deploying the object-file model: 0, 0 0, and 0 0 0 are the representations of 1, 2, and 3 objects respectively. Further, infants compare models on the basis of 1:1 correspondence, represent ordinal relations among sets of 1, 2, and 3 objects, and know that (0) plus another object yields (0 0), and that (0 0) plus another object yields (0 0 0). (Empirical support for these propositions was provided in an earlier section.) Second, the child learns the integer list as a meaningless ordered list of words and the counting routine as a number-irrelevant game like patty-cake. In the course of playing the counting game, the child learns first that "one" applies to arrays in which just one object file is open (0), next that "two" applies to arrays in which two object files are open (0 0), and next that "three" applies to arrays in which three object files are open (0 0 0).

The stage is set, now, for the crucial induction, which begins by the recognition of an analogy. The child notices an analogy between two different "follows" relations-the relation of (-0) between (0). (0 0), and (0, 0, 0), instantiated by the opening of a new object file, and the relation of *follows* in the count list. This analogy licenses the crucial induction: if x is followed by)' in the counting sequence, opening a new object file in what is called an x array results in what is called a *v* array. This proposal does not yet embody the arithmetic successor function. but one additional step is all that is needed. Since the child has already mapped single object files onto "one," opening a new object file is equivalent to *adding one.*

Lessons from the study of the development of number representations: Structural analogy

I began with evidence that prelinguistic infants represent number, that they can carry out computations that establish numerical equivalence, establish ordinal relations between the numerosity of two sets, and compute the outcome of adding or subtracting one from small sets. These data fall on the side of positions that posit rich initial representations as the building blocks of cognition. But before such behavioral data can inform debates at the level of structural analogy between cognitive and neurobiological mechanisms, it is important to discover the nature of the representational systems that subserve behavior on these tasks.

Infants most probably deploy two distinct systems in tasks that require representation of number-object-file and analog-magnitude representations. Further. I sketched some respects in which each is very different from, and weaker than, the explicit integer-list representation of number that is mastered in the child's fourth year of life. The acquisition of the integer-list representation of number thus constitutes an example of a developmental discontinuity. I concluded with a speculative sketch of bootstrapping mechanisms, illustrating with an example of a bootstrapping process that might achieve the new representational resource. These latter arguments fall on the side of constructivist accounts of the creation of new representational resources during development.

At the level of structural analogy, then, I have argued that constructive processes, rather than selective ones, underlie the acquisition of the integer-list representation of number, and that. this process involves the creation of a representational resource with more power than those from which it is built. As far as I am aware, there are no known neural developmental processes analogous to the bootstrapping mechanisms sketched above. For example. the activity-driven processes that are involved in dendrite growth that Quartz and Sejnowski (1997) offer as an example of constructive processes provide only a very distant analogy to those that are involved in the case of cognitive development detailed here. How might we go further in closing the gap in this case?

Closing the gap: Integrating cognitive development with systems neuroscience

Detailed models of representational format play two roles in the argument I am developing here. Heretofore, my main point has been that it is only by examining the initial representational systems in detail that we can say in what *ways* they are continuous and in what ways discontinuous with later representational systems, and only then can we motivate possible learning mechanisms that take the initial systems as input and output representational systems that transcend them. Here, I wish to emphasize a second point: It is the same level of detail (at least) that is needed to begin to close the gap between characterizations of the developing mind and characterizations of the developing brain.

I have provided evidence for two systems of numerically relevant representations (object-file representations, analog-magnitude representations) that become integrated, during development, in the creation of a third system-the explicit integer-list representation that supports counting. This work motivates several questions at the level of systems neuroscience. Are there distinct neural circuits for these computationally distinct representational systems? Do they continue to articulate mathematical reasoning into adulthood? Existing evidence suggests an affirmative answer on both counts (for reviews, see Dehaene, 1997; Butterworth, 1997).

The object-file system was originally studied in the context of object-based attention, and it clearly continues to articulate midlevel vision in adulthood. Data from studies of patients with neuropsychological damage and from imaging studies converge on the conclusion that parietal cortex, especially inferior parietal cortex, is part of the neural circuit underlying midlevel object-based attention. For example, Culham, Brandt, Cavanagh, Kanwisher, Dale, and Tootell (1998) found that activity levels in this area were correlated with the number of objects being simultaneously being tracked in the Pvlvshvn multiple object tracking task (other areas. including regions of frontal cortex, also showed this relation). Parietal damage is also implicated in object-based neglect (e.g., Behrman and Tipper, 199-l) and the even more striking Balint syndrome (for a review, see Rafel, 1997). Patients with Balint syndrome exhibit simultanagnosia, the incapacity to perceive two objects at once, in the face of normal sensory processing. They cannot do simple tasks such as telling which of two lines is longer, and do not report seeing two simultaneously presented disks, although if the disks are joined by a bar, making a dumbbell, they report seeing the whole dumbbell (Luria, 1959). It seems likely that these svndromes reflect damage to the midlevel object-file/object indexing attentional systems (for development of this point, see Scholl, in press).

The object-file system, with its capacity to simultaneously index and track up to 4 objects, is not a dedicated number representational system. Number is merely implicitly represented here; the objects indexed constitute a set number that is represented by one object-file for each object in the set. The analog-magnitude representational systems, in contrast, represent specifically *number.* There are analog-magnitude representations for other physical quantities, of course, such as distance, size, and time; but the computations that establish representations of number in a model such as Meck and Church's create an analog symbol for number alone. Dehaene (1997) has summarized the evidence that analog-magnitude representations of number are implicated when adults compare numerical quantities and carry out some arithmetical operations (addition and subtraction). Furthermore, there is very strong evidence that the inferior parietal cortex is also a crucial part of the circuit that subserves the mental representation of approximate numerical quantities.

As Dehaene reviews, extensive neuropsychological data indicate that damage to the inferior parietal cortex of the dominant hemisphere impairs comprehending, producing, and calculating with numbers; and damage to these areas early in development results in Gerstmann's syndrome, a permanent disability in calculation. Sometimes the damage is selective for calculation, sparing reading, writing, spoken recognition, and production of Arabic digits and numerals. Dehaene (1997) argues that the core deficit is due to the disorganization of the analog-magnitude representation of numerical quantities, rather than calculation per se. He presents a case, Mr. Mar, who makes 75% errors on single-digit subtraction (e.g., 3 - 1), and 16% errors judging which of two numbers is larger. Most tellingly, Mr. Mar makes 77% errors in saying what number comes between two other (e.g., 7, 9), in spite of being flawless at saving which letter comes between two (b, d), which day comes between two (Tuesday, Thursday), and which month comes between two (March, May). In both PET and fMRI studies, the same area is active during simple calculation, and activity during number comparison tasks reflects difficulty as reflected in the Weber fraction (Dehaene, 1997) .

In this chapter I have stressed the functional differences between the small number, object-file, system and the analog-magnitude system. As just reviewed, parietal regions are centrally involved in both. Within-subject studies are needed to establish whether different parietal areas are differentially implicated in the two sorts of tasks.

Dehaene (1997) makes it clear that at least three separate representations of number are involved in adult number processing. In addition to the circuit that includes parietal number sense, there is a separate system that represents visual Arabic numerals and involves the ventral occipitotemporal section of both hemispheres, and also a system that represents the verbally coded integer list, which, like any verbal representation, crucially implicates left inferior temporal cortex. Dehaene points out that each of these different systems supports different calculations; the analog-magnitude representations are automatically recruited in number comparisons and in simple calculation, and in calculations involving approximate representations, whereas verbal systems support multiplication and exact calculation that draw on memorized number facts.

This merest sketch allows us to explore how to begin to close the gap between the description of the acquisition of number representations culled from cognitive studies and developmental neuroscience. It seems likely that the infant's numerical competence is subserved by the circuits that include parietal cortex, those that play a role both in object-file representations and analog-magnitude representations. And it certainly is the case that the integer-list representation of number constructed during the preschool years is subserved by the circuits that include inferior temporal cortex. Presumably, the initial representation of the list of words as a meaningless ordered list also recruits these areas. So how are we to think, in neurological terms, about the processes that construct *numerical* meaning for that verbally represented list? At the very least, we would want to know how the brain constructs mappings between different representational systems. As argued above, in this case the mapping process is not merely a process of association. This is the principal challenge that tuiderstanding of the development of number representations at the behavioral and computational levels puts to developmental cognitive neuroscience.

One test that meaningful connections between the neural and cognitive/computational levels of description are being forged is that data about the developing brain can be brought to bear on controversies formulated at the more abstract level of description. An example alluded to earlier was the Diamond and Goldman-Rakic work suggesting that A/not B errors on Piagetian object permanence tasks reflect frontal lobe immaturity rather than lack of object representations. I conclude my case study of number representation with an illustration of how developmental cognitive neuroscience studies might resolve an important question concerning the development of number representations.

As Dehaene (1997) summarizes, there is no doubt that in the adult state the analog-magnitude representations of number are integrated with both the verbal integer list and the written Arabic numeral representations, andd are activated automatically during number comparison tasks and certain arithmetic calculations. In some real sense, then, these evolutionarily ancient and early developing nonverbal representations ground number understanding throughout the lifetime. But are they the ontogenetic roots of explicit number representation? Do they ground the initial forming of the integer-list representation? The bootstrapping proposal presented above had no role for analog-magnitude representations of number in the process through which toddlers build the integer-list representation of number. If that proposal is on the right track, two very important questions become *when* and *how* does the integer-list representation become integrated with analog-magnitude representations? Although purely behavioral data will bear on these questions, imaging studies with preschool-aged children, should the techniques become available, also will provide invaluable information.

Final conclusions

We are far from being able to say anything about the growth of the patterns of neural connectivity that underlie the development of number representations described in this chapter. I have argued that characterizing the details of representational format and the computations defined over those representations, as well as characterizing development at these levels, is a necessary prerequisite for such understanding. As always, an understanding of the mind must guide the search for its neural underpinnings.

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