

On Learning New Primitives in the Language of Thought: Reply to Rey

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Abstract: A theory of conceptual development must provide an account of the innate representational repertoire, must characterize how these initial representations differ from the adult state, and must provide an account of the processes that transform the initial into mature representations. In Carey, 2009 (*The Origin of Concepts*), I defend three theses: 1) the initial state includes rich conceptual representations, 2) nonetheless, there are radical discontinuities between early and later developing conceptual systems, 3) Quinean bootstrapping is one learning mechanism that underlies the creation of new representational resources, enabling such discontinuity. I also claim that the theory of conceptual development developed in *The Origin of Concepts* addresses two of Fodor's challenges to cognitive science; namely, to show how learning could possibly lead to an increase in expressive power and to defeat Mad Dog Nativism, the thesis that all concepts lexicalized as mono-morphemic words are innate. A recent article by Georges Rey (*Mind & Language*, 29.2, 2014) argues that my responses to Fodor's challenges fail, because, he says, I fail to distinguish concept possession from manifestation and I do not confront Goodman's new riddle of induction. My response is to show that, and how, new primitives in a language of thought can be learned, that there are easy routes and hard ones to doing so, and that characterizing the learning mechanisms involved is the key to understanding both concept possession and constraints on induction.

Alone among animals, humans can ponder the causes and cures of pancreatic cancer and global warming. How are we to account for the human capacity to create concepts such as *electron*, *cancer*, *infinity*, *galaxy*, and *democracy*? Rightly, most attempts to provide such an account center on specifying what makes concept attainment possible, but the literature on concept development adds a second question. Why is concept attainment (sometimes) so easy and what (sometimes) makes concept attainment so hard? Easy: some new concepts are formed upon first encountering a novel entity or hearing a new word in context (Carey, 1978). Hard: others emerge only upon years of exposure, often involving concentrated study under metaconceptual control, and are not achieved by many humans in spite of years of explicit tutoring in school (Carey, 2009). As we will see, considering what underlies this difference illuminates the nature of concept attainment.

A theory of conceptual *learning* must have three components. First, it must characterize the innate representational repertoire—that is, the representations that subsequent learning processes utilize. Second, it must describe how the initial stock of representations differs from the adult conceptual system. Third, it must

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characterize the learning mechanisms that achieve the transformation of the initial into the final state. *The Origin of Concepts* (henceforth *TOOC*) defends three theses. With respect to the initial state, contrary to historically important thinkers such as the British empiricists, Quine, and Piaget, as well as many contemporary scientists, the innate stock of primitives is not limited to sensory, perceptual or sensory-motor representations; rather, there are also rich innate conceptual representations with contents such as *object*, *agent*, *goal*, *cause*, and *approximate cardinal value of a set of individuals* (part of what makes the human conceptual repertoire possible). With respect to developmental change, contrary to ‘continuity theorists’ such as Fodor, Pinker, Rey and others, conceptual development involves qualitative change, resulting in systems of representation that are more powerful than, and sometimes incommensurable with, those from which they are built (part of what makes attaining the human conceptual repertoire sometimes hard). With respect to a learning mechanism that achieves conceptual discontinuity, I offer Quinean bootstrapping (part of what makes attaining the human conceptual repertoire possible.)

While the goal of *TOOC* was to explicate and defend these three theses I also addressed Fodor’s (1975, 1980) two related challenges to cognitive science—first, to show how learning can possibly result in increased expressive power, and to defeat the conclusion that all or almost all mono-morphemic lexical concepts are innate. The key to answering both of these challenges, as well as to understanding conceptual discontinuities in general, is to show that, and how, new conceptual *primitives* can be learned. Conceptual primitives are the building blocks of thought, the bottom level terms that articulate mental propositions and otherwise enter into inference.

Rey (2014) denies that the project is successful in meeting Fodor’s challenges. Although I ultimately disagree, I appreciate many of the points Rey makes along the way. Rey’s discussion brings into focus how the projects of understanding conceptual development and understanding the nature of concepts, learning, and the human mind are intertwined.

1. The Dialectic According to Rey

A kind of logical constructivism is at the heart of Fodor’s and Rey’s dialectic. They take expressive power to be a function of innate primitives, and what can—in principle if not in fact—be built from them using the resources of the logic available to the learner. Thus, expressive power, Rey says, is a logical/semantic notion. So long as learning mechanisms are characterized solely by the set of primitives and the logical resources through which one composes new representations in terms of primitives, clearly one cannot increase expressive power by learning (Fodor, 1980).

My response to this picture of learning and conceptual development is to argue that learning mechanisms can create new primitives, new primitives that cannot be defined in terms of antecedently existent primitives, and thus increase the

expressive power of the conceptual system. In addition, my concern is with how new primitives actually come into being; if there are processes that yield new primitives that do not involve logical construction, then the question is whether such processes actually underlie the emergence of any given representation.

Fodor's (1975) second challenge to cognitive science is to defeat his argument for Mad Dog Nativism, that is, to defeat the argument that virtually all of the over 500,000 mono-morphemic concepts lexicalized in the Oxford English Dictionary are innate. Rey lays out Fodor's argument as follows:

- Premise 1: (CONFIRMATION) All learning is hypothesis confirmation.
- Premise 2: (LOGICAL CONSTRUCTION) One can learn new concepts only by creating and confirming hypotheses formulated in terms of logical constructions from antecedently available primitive concepts.
- Premise 3: (ATOMISM) Mono-morphemic cannot be analyzed as logical constructions of other concepts primitive or otherwise. (Actually, Fodor says 'most' mono-morphemic concepts cannot be so analyzed, but for simplicity I will assume 'all' rather than 'most'.)
- Conclusion: (INNATENESS) In order to acquire a new mono-morphemic concept, one would have to confirm hypotheses already containing the mono-morpheme (i.e. already containing the concept to be learnt). Therefore, no mono-morphemic concept can be learnt.

Rey rightly says that *TOOC* answers this challenge by giving reasons to deny Premises 1 and 2, CONFIRMATION and LOGICAL CONSTRUCTION. My basic strategy has been to provide several case studies of transitions between conceptual systems in which the later one expresses concepts that are not logical constructions from the earlier one (Carey, 1985, 2009; Smith, Carey, and Wisner, 1985; Wisner and Carey, 1983). Sometimes this is because of local incommensurability, as in case studies of thermal concepts, biological concepts and electromagnetic concepts in the history of science, or concepts of matter/weight/and density in intuitive physics in childhood and the concepts of life and death in childhood). Sometimes it is because of developments within mathematic representations that increase expressive power without involving local incommensurability (as in case studies of the origins of concepts of integers and rational number).¹ *TOOC* then goes on to analyze how Quinean bootstrapping plays a role in transitions of both types.

Rey argues that *TOOC* fails to answer Fodor's challenge for two interrelated reasons: it fails to distinguish between concept possession and concept manifestation, and second, whenever Quinean bootstrapping involves inductive inference, it fails to confront Goodman's new riddle of induction. In response, I reply that Rey's assumptions about concept possession are empirically wrong, as they presuppose

¹ The case study of the construction of the integers is the focus of Rey's commentary. I will discuss whether this episode of conceptual development truly involves a discontinuity, and an increase of expressive power, when I turn to it in Sections 7 through 12 below.

Fodor's Premises 1 and 2, and that the explanatory goals of Quinean bootstrapping do not require any consideration of the justification of induction.

As Rey explicates the distinction between possessed and manifest concepts, a concept is manifest if it is currently available to support thought, learning, inference and action. He notes, correctly, that relatively few of the 500,000 mono-morphemic concepts lexicalized in English are manifested in the infant's mind. A concept is possessed if it has the potential to be manifest. This technical notion, 'possession', is unproblematic, but empty. It has the consequence that even extreme anti-nativists would have to agree that all concepts that are ever manifest are possessed at birth. Nobody would ever deny that an actual manifest concept had the potential to be the output of some developmental process. This is bizarre terminology, and for this reason, psychologists do not talk about 'concept possession' with this meaning. However, we *do* explore the possible outputs of the learning mechanisms we investigate, and indeed, such exploration is an important part of characterizing them. Rey then goes on to make unsupported (and often wildly misleading) assumptions about concept possession. He assumes that possessed concepts constitute an innate space of alternatives, laying in wait to become manifest, and that manifestation consists in *being logically constructed* from these innately possessed concepts, or that manifestation is some unspecified process through which an already existing unmanifest but possessed innate concept becomes manifest and thus available for thought. These are substantive claims, claims my research aims to address.

Rey argues that the distinction between manifest and possessed concepts crucially matters to the Fodorian dialectic. He points out expressive power is a semantic/logical issue (i.e. what concepts can be definitionally constructed, given specific assumptions about innate primitives and innate logical machinery). As such, he says, any data that provides information about which concepts are manifested at any given time are simply irrelevant to the question of expressive power. Therefore, the data in support of conceptual discontinuities in manifested concepts do not provide evidence for increases in expressive power. Expressive power, Rey claims, is properly seen as a question about concept possession, not concept manifestation.

Rey concludes with an ecumenical proposal that he fears will please neither me nor Fodor. Paraphrasing and perhaps putting words into his mouth: the process of coming to *manifest* a concept may be an intentional process worthy of psychological research, may involve learning, may involve discontinuities (in *manifest* concepts), may even sometimes require bootstrapping (Quinean or otherwise). That is, bootstrapping may sometimes be part of the process through which possessed concepts become manifest, but it does not and cannot yield representations that are genuinely new.

2. Responding to Rey

Contrary to Rey, the only question of expressive power that makes any sense concerns manifest concepts. I embrace the proposal that there is a worthy psychological

project concerning the acquisition of manifest concepts, although, of course, I think that bootstrapping provides us with new manifest conceptual primitives and so increases in expressive power. The developmental primitives I study are those we can find evidence for in the baby's or animal's behavior. They must be available to support inference and action in order to be diagnosed, i.e. they must be manifest (currently available for thought). Not only do I think there *may* be a project worthy of psychological study here, the burden of my long book is to reveal the fruits so far of this study. In what follows I argue that concept manifestation is where the debates about expressive power, conceptual continuity/discontinuity, and induction *actually* play out. That is, it is the expressive power of *manifest* concepts that concerns me.

To gain an initial appreciation of these points, consider Rey's unproblematic, but empty, characterization of possessed concepts. One possesses a concept if one could, at the end of a process of development, end up using that concept in thought. One manifests a concept if one can use it now. On this sense of possession there simply is no debate whether *seven* or *quark* or *democracy* are innately possessed, for these *do* arise at the end of some process of development. This cannot be what the debate is about. The debate becomes substantive only in the face of actual proposals about learning mechanisms and innate primitives (e.g. Fodor's Premises 1 and 2). The input to any given learning process (hypothesis testing or otherwise) consists of manifest representations. If learning is hypothesis testing, it can only involve choice between manifest concepts.

My project is to account for the acquisition of manifest mental representations. Rather than being irrelevant to the question of expressive power, questions of what the innate manifest primitives are, what the actual computational mechanisms that constitute learning are, are central to the project of exploring expressive power, in the logical/semantic sense. One can only explore the logical/semantic expressive power of an innate conceptual system relative to proposals for *actual* innate primitives and *actual* learning mechanisms.

Expressive power, as Rey is using the notion, is the set of concepts that *could* be defined, using innate primitives and combinatorial machinery. This should be stated: 'innately manifest primitives and *innately manifested* combinatorial machinery' for these are the representations and computational devices available for learning. At this point in the dialectic Rey moves from the unproblematic, but empty, notion of possession to his substantive views about the initial state. What the innate primitives are and what the innate combinatorial machinery is are empirical questions about innate representational machinery, and these questions about expressive power concern innately manifested representations.

For any representational system we posit, we are committed to there being answers to three questions. First, what is the format of the symbols in the system; second, what determines their referents; and third, what is their computational role in thought. A worked example in *TOOC* is the evolutionarily ancient system of number representations in which the mental symbols are quantities (rates of firing, or size of populations of neurons) that are linear or logarithmic functions of the cardinal values of sets, which in turn are input into numerical computations

such as number comparison, addition, subtraction, multiplication, division, ratio calculations, probability calculations, and others (see Dehaene, 1997, for a book-length treatment of this system of numerical representations). We can only explore such systems with psychological methods that diagnose manifest representations. The project of *TOOC* is understanding the representational resources available as the child or adult interacts with the world, how these arise and change over development. These representations are the ones available for hypothesis testing, as input into further learning, and to play a computational role in thought. And it is successive manifest conceptual systems one must analyze to establish qualitative change.

In what follows I flesh out these points, bringing out what I consider to be the real issues Rey raises concerning how *TOOC* attempts to answer Fodor's challenges to cognitive science. The real issues include a characterization of the nature of learning (Fodor's first premise), the unjustified acceptance of the logical construction model as the only model of concept learning (Fodor's second premise), the misleading analogy of the totality of concepts ultimately attainable as a hypothesis space, the characterization of how primitives arise (both in cases where this is easy and in cases where this is hard), and the characterization of constraints of induction (and learning more generally, in cases where learning does not involve induction).

I begin with the premises in Fodor's argument that I deny. I first comment on why these premises matter and I then show why they are wrong.

3. Premise 2. Logical Construction

TOOC is an extended argument against Premise 2 of Fodor's argument: the view all concepts must either be innate or definable from innate primitives through innate logical combinatorial devices. The logical construction premise is widely adopted within cognitive science. Dominant views of concept acquisition—the classical view, prototype theory, exemplar theories, and internalist versions of the 'theory-theory' or 'knowledge views'—all presuppose it (Smith and Medin, 1981; Murphy, 2002; *TOOC*). On all these views, new concepts are composed from primitives using logical/syntactic combinatorial devices: 'barks and wags tail when happy and . . .' for the prototype of a dog for example. The issue here is the nature and origins of newly created concepts, not how they determine category membership (necessary and sufficient conditions, probabilistically). The dominant theoretical project within the field of lexical development in the 1970s was to attempt to discover the lexical primitives in terms of which lexical items are defined, and to study the intermediate hypotheses children entertain as they construct new concepts from those primitives (see Carey, 1982, for a review and critique). There I called this view 'piece by piece construction'; Margolis and Laurence (2011) call it 'the building blocks model'. Here, I will call it 'the logical construction model', in honor of Premise 2. In contrast, I argue (Carey, 1982, *TOOC*) that computational

primitives need not be innate. They can be acquired through learning processes that do not consist of logical construction from innate primitives.

As Rey points out in the end of his article, one central issue is atomism. If many of the primitives in adult thought (e.g. lexical concepts like ‘dog’ or ‘cancer’), cannot be defined in terms of innate concepts, then they either must be innate primitives or it must be possible to learn primitives through some mechanism that does not consist of defining new concepts by logical combination of antecedently available terms. I accept Fodor’s doctrine that most lexical concepts are computational primitives.

Notice that the possibility of learning new primitives matters to the question of expressive power of the system, in the logico/semantic sense. The expressive power of a system of representations is a function of its atomic terms and combinatorial apparatus. The logical connectives and operators (e.g. sentential operators, modals, quantifiers) are not the only primitives that matter to expressive power. If ‘dog’ cannot be logically constructed from primitives, then acquiring a concept ‘dog’ increases expressive power of the system (see Weiskopf, 2007). That is, non-logical primitives figure into semantic/logical expressive possibilities as well as do logical ones. This is one reason that the question of whether one learns the concept *dog* is so central to the debate between Fodor and his critics.

4. Premise 1. All Learning is Hypothesis Formulation and Testing

As Margolis and Laurence (2011) point out in a reply to Fodor’s 2008 book (*LOT2*), a cursory examination of the variety of attested learning mechanisms in the animal kingdom shows this generalization to be wildly off the mark. Rote learning (memorizing a phone number), one-trial associational learning (e.g. the Garcia effect, the creation of a food aversion as a result of becoming nauseous some fixed time after having eaten a novel food: Garcia *et al.*, 1955), and many other types of learning do not involve choosing among multiple hypotheses, confirming one of them. And as we shall see, such mechanisms have roles to play in creating new conceptual primitives.

Of course, the claim that these are learning mechanisms depends upon what one takes learning to be. Learning mechanisms share a few essential properties that allow us to recognize clear examples when we encounter them. All learning results in representational changes in response to inputs that can be seen (by the scientist) to provide evidence relevant to the representational change. That is, learning is a computational process, requiring inputs that can be conceptualized as providing relevant *information*. Sometimes, as in the case of explicit or implicit hypothesis testing, the organism itself evaluates the information in the input with respect to its evidential status (as in all forms of Bayesian learning mechanisms). But other times, the learning mechanism is a domain-specific adaptation that responds to information by simply effecting a representational change of relevance to the organism—an example being the learning mechanism that underlies the Garcia effect mentioned above.

5. The Relatively Easy Route to New Representational Primitives: Domain Specific Learning Mechanisms

The issues concerning possession and manifestation, learning and acquisition, arise in the case of any representation, conceptual or otherwise, that end up in the repertoire of an animal. The literatures of psychology and ethology have described hundreds of domain-specific learning mechanisms that simply compute new representations from input, having arisen in the course of natural selection to do just that. Most of these representations are not conceptual ones, but considering how they are acquired shows that Fodor's and Rey's assumptions about learning do not hold in general. These learning mechanisms do not involve hypothesis testing, and thus provide counterexamples to Premise 1. They also do not implement logical construction from primitives, and thus provide counterexamples to Premise 2. Considering how they work illuminates why it's a mistake to consider possessed representations (those with the potential of becoming manifest) as a space of existent representations, ready to be chosen among or built from in a process of manifestation.

TOOC's example of an evolved domain-specific learning mechanism is that which underlies Indigo Buntings' *learning* which part of the night sky indicates north. This matters crucially to Indigo Buntings, for they migrate over 3500 miles each spring (north) and fall (south), and they navigate by the stars. Because the earth tilts back and forth on its axis, what part of the night sky indicates north changes radically on a 30,000 year cycle. Sometime not too far in the future, the North Star will be Vega, not Polaris. Thus, it is unlikely that an innate representation of Polaris as the North Star was created by natural selection, and indeed, Steven Emlen (1975) discovered the learning mechanism through which Indigo Buntings create the representation of north that will play such a crucial role in their migratory life. The learning device that achieves this analyzes the center of rotation of the night sky, and stores the configuration of stars that can allow the bird to recognize the position of north from a static sighting (as it has to do every time it starts to fly during its migrations in the spring and the fall, and as it monitors its course).

This mechanism computes what it is designed to compute—nothing more nor nothing less. It creates an essential representation in the computational machinery of Indigo Buntings, namely a symbol held in long-term memory that specifies north in the night sky. Of course, there is a prepared computational role for this representation, but the representation of north in the night sky must still be learned. Domain specific learning mechanisms of this sort are often supported by dedicated neural machinery that atrophies after its work is done, leading to critical periods. This is such a case; if a bird is prevented from seeing the night sky as a nestling, no amount of exposure to the rotating night sky later in life allows the bird to identify north, and the bird perishes.

This example is worth dwelling upon with respect to the distinction between possession and manifestation, whether possessed concepts should be thought of as an existing space of hypotheses, and whether the mechanism involves hypothesis formation and confirmation. Take possession versus manifestation first. Until the

learning episode is completed, there is no manifest representation that specifies what direction is north in the bird's mind. However, this learning mechanism can learn any of a very large number of star configurations constellations that could indicate 'north'. Indeed, part of the evidence that this *is* the learning mechanism through which Indigo Buntings establish Polaris as the North Star are planetarium experiments in which the night sky is made to rotate around an arbitrarily chosen part of the night sky while the birds are nestlings. Indeed, the birds then use the North Star so specified to set their course when it's time to migrate. Thus, in the potential sense of possession, there are a plethora of representations of 'north'. And clearly, one can investigate limits on the system (e.g. if stars were equally distributed throughout the sky, or if they were too densely packed to be resolved, or if the patterns of stars showed large scale repetitions, this couldn't work.) This is how one would explore the space of possessed representations in this system, in the sense of possible representations it can achieve—the logical space of possibilities for specifying an absolute direction by the celestial navigation system of actual Indigo Buntings. But there is no positive characterization anywhere in the birds' mind of any of the possibilities (including Polaris). There are no possessed representations that in any way exist in the Bunting neonate's mind. Also we see that it is only with an actual representational/computational characterization of this learning mechanism that the logical space of the possessed concepts (in the sense of those it is possible for the Bunting to learn) can be explored. Such is always the case.

What about hypothesis testing? I take the essential features of hypothesis testing to be two: the learning mechanism must entertain alternatives, and choose among them on the basis of evidence. In no way does Indigo Buntings' acquiring a representation of north consist of choosing among possibilities. The animal doesn't consider any other than the one that the output of the learning mechanism. Here, calling the possible specifications of north a 'hypothesis space' is wildly misleading.

This case is also worth dwelling upon with respect to the other issues on the table. Not only does this case not involve hypothesis formulation and testing, it also does not involve building a new representation out of primitives by logical combination. And since there is no induction involved, the issues of constraints on induction do not arise. Of course, all learning mechanisms must be highly constrained to be effective, and characterizing real learning mechanisms allows us to understand the constraints under which they operate. This is a highly constrained learning mechanism; it considers only some kinds of information to create a representation that has only one computational role. It is of no use to the bird in helping the bird learn what to eat, who to mate with, or where its nest is in a local environment.

Navigation is not a special case. There have been hundreds of such domain-specific learning mechanisms detailed in the literatures of ethology and psychology, including the imprinting mechanisms that allow infants (animals and humans) to identify conspecifics in general and their caretakers in particular, mechanisms that allow animals to learn what food to eat (the Garcia effect just one of dozens of domain-specific learning mechanisms through which omnivores like rats and humans achieve this feat), bird song learning, and so on (see Gallistel *et al.*, 1991, for

a review of four such domain-specific information-expectant learning mechanisms, and Gallistel, 1990, for a nuanced discussion of the nature of learning).

In sum, the animal literature provides many examples of learning mechanisms designed to form new computational primitives, learning mechanisms that implicate neither logical construction from existing primitives (Premise 2), nor hypothesis testing and confirmation (Premise 1). One can (and one does) explore the space of possible outputs of these mechanisms, for this is one way they can be fully characterized and their existence empirically tested, but in no way are the ‘possessed’ concepts laying in wait, existing ready to be manifested.

6. The Relatively Easy Route to New Conceptual Primitives

The learning mechanisms described above acquire new primitive representations, but they are probably best thought of as new perceptual rather than *conceptual* representations, as their computational role is sensori-motor. There are, however, learning mechanisms that similarly respond to inputs of certain types by simply creating new conceptual primitives, conceptual in the sense of representations used in thought and reasoning. These domain-specific concept learning mechanisms need not involve hypothesis testing, and do not involve constructing new concepts by logical combination. Take the Block (1986)/Macnamara (1986)/Margolis (1998) object-kind learning mechanism for example.² This learning mechanism is triggered by encountering a novel object (as specified by core cognition of objects) with obviously non-arbitrary structure. As Prasada *et al.* (2002) showed, there are several cues to non-arbitrary structure: the object has complex yet regular shape (e.g. symmetries, repetition), or there are multiple objects that share a complex irregular shape, or the object has functionally relevant parts, or the object recognizably falls under an already represented superordinate kind (e.g. kind of agent, kind of animal, kind of artifact). Core cognition contains perceptual input analyzers that are sensitive to cues to each of these properties of individual objects. Encountering an individual with one or more of these properties triggers establishing a new representational primitive that can be glossed *same basic level kind as that object*. Reference to the kind is ensured by representation of the surface properties of the individual or individuals that occasioned the new concept (and these represented surface properties get enriched and even overturned as bases of reference and categorization as more is learned about the kind). The content of the new concept depends upon the

² These writers discuss this mechanism as a *natural kind* learning mechanism (e.g. kinds of animals), but I believe the domain of this mechanism is object kind representations (as opposed to object properties, individual objects, or the events in which objects participate). Roughly, kind representations are inductively deep, and kinds are construed in accordance with the constraints that constitute psychological essentialism in Streven’s (2000) sense. Artifact kinds fall under the domain of this mechanism as well as do natural kinds (Kelemen and Carey, 2007).

referent, the conceptual role provided by the basic level kind schema (psychological essentialism), and the conceptual roles provided by any superordinate kind schemas that the individual is taken to fall under (e.g. *agent*, *animal*, *artifact*, these in turn being constrained by their roles in different systems of core cognition or constructed theories).

Consider encountering a kangaroo for the first time. Such an encounter might lead to the formation of a concept *kangaroo* that represents animals that are the same basic level kind as the newly encountered one. No enumerative induction is needed; the concept is what Strevens (2012) calls ‘introjected’ into one’s set of primitives. This concept, falling under psychological essentialism (as it is a kind concept), reflects the many constraints on kind concepts. That is, the conceptual role ‘same kind as’ includes assumptions that something causes the non-random structure that triggered the formation of the new concept, that these underlying causes are shared by all members of the kind (now, in the past, in the future), that the surface properties that specify the individual that occasioned the new concept may not hold for all members, possibly not even typical members. Furthermore, the current guesses about the nature of the relevant causal mechanisms relevant to the creation of members of this kind, to determining their properties, and to tracing numerical identity though time, are taken to be open to revision. That is, there is no definition that determines membership in the kind; learners treat everything they represent about the kind up for revision (including, even that there IS a new kind—the individual we encountered might have been a mutant raccoon).

This mechanism creates new primitives, not definable in terms of other manifest concepts, and thus increases the expressive power of the conceptual system. The concept *kangaroo* is not definable in terms of antecedently available primitives using the combinatorial machinery of logic. Before creating this concept, one could not think thoughts about kangaroos, just as before analyzing the center of rotation of the night sky and storing a representation of *north* so specified, an Indigo Bunting could not set or guide a course of flight according to *toward north* or *away from north*. Of course the kind learning mechanism ensures that creating new primitives for kinds is easy; one need only encounter an individual that one takes to be an individual of a new kind, and store a representation of what that individual looks like. But this process involves neither induction nor hypothesis testing among a huge space of possessed but unmanifest concepts. The concept *kangaroo* was not laying in wait in a system of representations available for selection by a Bayesian hypothesis testing mechanism.

Rey discusses the Margolis kind learning module, claiming that it falls prey to Goodman’s ‘grue’ problem, just as Quinean bootstrapping does (see below). There are two answers to Rey’s questions regarding constraints on induction in the Margolis kind learning module. First, as detailed above, there need be no induction. But, Rey asks, why are not kinds such as ‘object’ ‘animal’, ‘agent’, subcategory of kangaroo’, ‘kangadile’ (kangaroo until year 2040, thereafter crocodile), or ‘undetached kangaroo part’, on an infinitude of other concepts possible glosses of

same kind as that object, rather than the kind 'kangaroo'? Why does the learner not form a concept of a particular individual (Oscar) instead of a kind?

Formulating the constraints that give a psychological explanation of why we actually project predicates like 'green' over predicates like 'grue' are part of the bread and butter research into cognitive development. In the case of dedicated concept learning devices such as the object-kind learning device, the empirical project is specifying the constraints under which the system operates. That is, the empirical project is to provide a positive characterization of attested learning mechanisms. That there is a dedicated kind concept acquisition device is an empirical discovery, and, like all learning mechanisms this one embodies strong constraints. It is a discovery that there is basic level in kind concepts, and it is a discovery that basic level kinds are privileged in kind concept learning (e.g. Rosch *et al.*, 1976). It is a discovery that kind representations embody constraints derived from causal/functional analyses (see the work on psychological essentialism and the psychology of a causal/explanatory core to kind concepts (S. Gelman, 2003; Keil, 1989; Ahn and Kim, 2000; Lombrozo, 2009; Strevens, 2000). And the existence and structure of systems of core cognition (*agent*, *object*), as well as innately supported systems of causal and functional analysis, are empirical discoveries, as is the fact that these constrain kind representations from early infancy (Carey, 2009). These constraints do not rule out *ever* entertaining concepts such as *animal*, *Oscar*, *puppy*, or *kangadile*. After all, some of them are themselves innately manifest (e.g. *agent*) and are drawn upon as important parts of the constraints on the kind module. That is, *agent* is the content of a superordinate kind that constrains a newly formed basic level kind concept that falls under it. Other concepts of kinds of objects, such as subordinate and superordinate kinds, as well as stage and phase sortals like *puppy* or *passenger*, are routinely manifested after basic level kind representations are formed (e.g. Hall and Waxman, 1993). Even outré concepts such as *kangadile* and *undetached kangaroo part* are obviously entertainable (after all, Goodman and Quine did so, and we all can join in). But these concepts simply are not the output of the dedicated basic level kind learning device discussed above.

Furthermore, the basic level kind module is not the only early emerging word learning mechanism. The child *can* also form a concept of the particular individual, *Oscar*. The positive characterization of the learning mechanism that yields representations of individuals is another story, one that has also been told (e.g. Bélanger and Hall, 2006).

The basic level kind learning module creates new primitive concepts. Before a person has formed the concepts *kangaroo* or *shovel*, or concepts of any of potentially infinite new kinds, he or she cannot think thoughts about the entities that fall under those concepts. This learning mechanism thus results in an increase in expressive power. However, like the cases of the non-conceptual dedicated learning mechanisms discussed above (those that yield representations of conspecifics, caretakers, the north star), there is an innately specified conceptual role for kind concepts, in this case given by the abstract concept *kind of object* (characterized by the schema of psychological essentialism) and by the schemas of superordinate kinds embedded in

core cognition and constructed theories that the learner assigns the new concepts to. Such already existing schema and conceptual roles are always part of the relatively easy route to new primitives.

7. The Relatively Hard Route to New Conceptual Primitives

Quinean Bootstrapping also creates new primitives, thus increasing the expressive power of the conceptual system. It differs from those learning mechanisms described above in that did not arise through natural selection to acquire representations of a particular sort. Rather, it is one of the learning mechanisms that underlie the creation of representational resources that are discontinuous with (in the sense of being qualitatively different from, being locally incommensurable with, the representations of the same domain that were their input). It creates new conceptual roles, rather than merely creating new primitives for which there were prepared conceptual roles (as in the case in the easy route to new primitives, see above). But once created, these new conceptual roles provide constraints on the concepts that will be learned, just as in the relatively easy route to new conceptual primitives.

TOOC takes a particular episode along the way to creating a representation of integers as a central worked example of conceptual discontinuity and of Quinean bootstrapping. This case study, the creation of the first representations of a small subset of natural numbers, is discussed in Rey's critique, so I use it here as well in my reply to Rey. As the focus of Rey's critique, he uses it to illustrate the supposed pitfalls of failing to distinguish possession from manifestation, as well as of failing to fully appreciate Goodman's new riddle of induction.

Rey says, and I agree, that expressive power is a semantic/logical issue. Examples of questions about expressive power relative to number representations include whether arithmetic can be expressed in the machinery of sentential logic—provably no—and whether arithmetic can be expressed in the machinery of quantificational logic plus the principle that 1–1 correspondence guarantees cardinal equivalence—provably yes, if you accept Frege's proof. As such, he claims that any data that provides information about which concepts are manifested at any given time are simply irrelevant to the question of expressive power. Rey claims that expressive power is properly seen as a question about concept possession, not concept manifestation. What Rey misses in this discussion is the exploration of expressive power with such proofs is relevant to the question of how arithmetic arises in development *only* against empirically supported proposals for what the innate numerically relevant innate primitives are, and what form innate support for logic takes. If arithmetic can be derived from the resources of logic alone (with no numerical primitives), this is relevant to the question of the origin of arithmetic in ontogenesis *only* if the relevant logical resources are innate, and in a form that would support the relevant construction. If primitives with numerical content are needed as well (e.g. the principle that 1–1 correspondence guarantees cardinal equivalence, or the concepts *one* and *the successor principle*), then one must account

for how these arise in development. *TOOC* provides evidence that these numerical concepts are not part of the child's innate endowment, and that they arise only after the bootstrapping episode in which the numeral list representation of number is constructed.

Nonetheless, Rey is correct that *TOOC* does not consider the form innate support for logic takes, and how logical resources arise in development. Indeed, I am acutely aware of this lacuna, and of its relevance to our understanding of numerical development. These questions have been the focus of research in my lab for the past four years, and will be so for the next decade at least. We do not yet have answers concerning the form innate support for logic takes. My current guess is that innate logic is largely implicit, embodied in computations, and that bootstrapping is needed before children create the logical resources needed for the mathematical construction of the integers from such primitives. After all, these constructions did not arise in mathematics until after 2000 years of development of formal logic. However, as I say below, my picture of the ontogenesis of concepts of integers would be falsified by the discovery of manifest representations with numerical content in addition to the three systems for which we already have empirical support.

Thus, I acknowledge that it is conceivable that Rey *could turn out to be right* (not that he provides a shred of evidence) that a full characterization of the initial state will reveal expressive power sufficient to express arithmetic. If so, I would certainly back away from my claims about this bootstrapping episode increasing expressive power, saying that my studies concern how arithmetic capacities *actually* become manifest in ontogenesis. After all, the latter is actually my concern. I am quite certain that children do not construct arithmetic as Dedekind/Peano or Frege did, and I favor my bootstrapping story about what children actually do. But, if numerical or logical primitives are needed that themselves arise as a result of bootstrapping processes, then my claims of increases in expressive power stand.

At any rate, the actual process through which representations of integers arise is an existence proof of the possibility that bootstrapping *can* yield new primitives. The case study of the ontogenetic origin of integer representations illustrates all three major theses in *TOOC*: the existence of conceptual rich innate representations, conceptual discontinuity, and Quinean bootstrapping.

8. Core Cognition of Number (Rich Innate Representational Resources; *TOOC*, Chapter 4)³

Core cognition contains two systems of representation with numerical content: parallel individuation of small sets of entities in working memory models, and

³ The evidence for central claims in *TOOC*, along with citations of relevant literature, can be found in the chapters flagged throughout the current text.

analog magnitude representations of number. Analog magnitude representations were briefly sketched in Section 2 above. They are analog symbols that approximately represent cardinal values of sets. One signature of this system of number representation is that magnitudes are compared to one another on the basis of their ratios, and thus discriminability accords with Weber's law (discriminability is better the smaller the absolute value of the quantity) and exhibits scalar variability (the standard deviation of multiple estimates of a given quantity is a linear function of the absolute value of that quantity.) Analog magnitude representations of number have been demonstrated in many animals (rats, pigeons, non-human primates) as well as in humans from neonates to adults.

Analog magnitude representations are the output of paradigmatic perceptual input analyzers, but the analog magnitude symbols for number that are produced are conceptual in the sense of having rich central conceptual roles, including the many different arithmetical computations they enter into, and the fact that they are bound to (quantify over) many types of individuals (objects, events, auditory individuals).

A second system of core cognition with numerical content, parallel individuation, consists of working memory representations of small sets of individuals (three or fewer). The symbols in this system represent individuals (e.g. 3 crackers is represented {*cracker*, *cracker*, *cracker*}, probably with iconic symbols for each cracker. Unlike the analog magnitude number representation system, parallel individuation/working memory is not a dedicated number representation system, nor are there any symbols that represent cardinal values (or any other quantifiers) in these models; there are only symbols for individuals. These models are used to compute total volume and area of the individuals, and are input into spatial and causal representations. The numerical content in the system of parallel individuation is entirely implicit; the symbols in the models stand in 1-1 correspondence with individuals in the sets modeled. This is ensured by computations sensitive to spatiotemporal cues to numerical identity. The system must determine whether a given individual is the same one or a different one from a previously viewed individual to determine whether to add another symbol to the model. Further implicit numerical content is embodied in some of the conceptual roles these models enter into. More than one model can be entertained at any given time, and models can be compared on the basis of 1-1 correspondence to establish numerical order and equivalence. Importantly, this system of representation implicitly represents *one*. There is no explicit symbol with the content *one*, but the system updates a model of a set of one when a numerically distinct individual is added to it, yielding a model of a set of two (and ditto for sets of two and three), and the system similarly updates a model if individuals are removed from it. There is a strict upper limit to the number of individuals that can be held in working memory at any given time: 3 for infants. This set-size limit on performance contrasts with the ratio limit on performance that characterizes analog magnitude systems.

The parallel individuation system is perception-like in many ways, especially if the symbols for individuals are indeed iconic, as I suspect. Nonetheless, the parallel

individuation models themselves are conceptual in that they are held in a working memory system that requires attention and executive function, and enter into many further computations in support of rich central inferential processes (e.g. reasoning about the actions of agents upon objects, functional analyses, causal analyses, as well as quantitative computations).

Systems of core cognition are not the only innate resources relevant to conceptual development. *TOOC* assumes also early linguistic resources, but makes no attempt to specify their exact nature (a topic for another book). And, as commented above, the nature of logical resources available to infants and toddlers is virtually unstudied. Particularly relevant for number representations are linguistic representations that underlie the meanings of natural language quantifiers. Number marking in language (quantifiers, determiners, singular/plural morphology) requires representations of sets and individuals, and provides explicit linguistic symbols with numerical content ‘a, all, some, most, many, few . . .’. *TOOC* reviews evidence that before age 2 children have mastered some of the basic syntax and semantics of natural language quantifiers, and that these linguistic structures provide important early constraints on the meanings of verbal numerals, via syntactic bootstrapping. (Herein lies the answer to Rey’s question of why numerals are not taken as proper names for sets; by the time children are learning words for verbal numerals, they identify proper nouns from syntactic context, and they have analyzed numerals as determiners or quantifiers, which gets them into the right part of semantic space.)

9. Conceptual Discontinuity (*TOOC*, Chapter 8)

There are two steps to establishing discontinuities in development. The first, most important, step is characterizing the nature and content of symbols in successive systems of representation: Conceptual Systems 1 and 2 (CS1 and CS2). These characterizations allow us to take the second step: namely, to state precisely how CS2 is qualitatively different from CS1. With respect to numerical content, there are three CS1s: analog magnitude representations, parallel individuation, and natural language quantification.

The substantive claims in *TOOC* are that these three systems of representation exist, have been characterized correctly, and are the *only* representational systems with numerical content manifest in infancy and the toddler years. *TOOC*’s picture of number development would be falsified if evidence were to be forthcoming for innate numerical representations in addition to those described above, or different from them. Indeed, one aim of my current work on the logical resources of infants and toddlers is to search for such evidence.

CS2, the first explicit representational system that represents even a finite subset of the positive integers, is the verbal numeral list embedded in a count routine. Deployed in accordance with the counting principles articulated by Gelman and Gallistel (1978), the verbal numerals implicitly implement the successor function,

at least with respect to the child's finite count list. For any numeral that represents cardinal value n , the next numeral in the list represents $n + 1$.

CS2 is qualitatively different from each of the CS1s because none of the CS1s has the capacity to represent any integers. The new primitives are the concepts 1, 2, 3, 4, 5, 6, 7, . . . , the concepts that underlie the meanings of verbal numerals. Parallel individuation includes no summary symbols for number at all, and has an upper limit of 3 or 4 on the size of sets it represents. The set-based quantificational machinery of natural language includes summary symbols for quantity—(e.g. 'some, all') and importantly contains a symbol with content that overlaps considerably with that of 'one' (namely, the singular determiner, 'a'), but the singular determiner is not embedded within a system of arithmetical computations. Also, natural language set-based quantification has an upper limit on the set sizes that are quantified with respect to exact cardinal values (*singular, dual, trial*). Analog magnitude representations include summary symbols for quantity that are embedded within a system of arithmetical computations, but they represent only approximate cardinal values; there is no representation of exactly 1, and therefore no representation of +1. Analog magnitude representations cannot even resolve the distinction between 10 and 11 (or any two successive integers beyond its discrimination capacity), and so cannot express the successor function. Thus, none of the CS1s can represent 10, let alone 342,689,455.

As required by CS2's being qualitatively different from each of the CS1s that contain symbols with numerical content, it is indeed difficult to learn. American middle-class children learn to recite the count list and to carry out the count routine in response to the probe 'how many', shortly after their second birthday. They do not learn how counting represents number for another 1 1/2 or 2 years. Young two-year-olds first assign a cardinal meaning to 'one', treating other numerals as equivalent plural markers that contrast in meaning with 'one'. Some 7 to 9 months later they assign cardinal meaning to 'two', but still take all other numerals to mean essentially 'some', contrasting only with 'one' and 'two'. They then work out the cardinal meaning of 'three' and then of 'four'. This protracted period of development is called the 'subset'-knower stage, for children have worked out cardinal meanings for only a subset of the numerals in their count list.

Many different tasks, which make totally different information processing demands on the child, confirm that subset-knowers differ qualitatively from children who have worked out how counting represents number. Subset-knowers cannot create sets of sizes specified by their unknown numerals, cannot estimate the cardinal values of sets outside their known numeral range, do not know what set-size is reached if 1 individual is added to a set labeled with a numeral outside their known numeral range, and so on. Children who succeed on one of these tasks succeed on all of them. Furthermore, a child diagnosed as a 'one'-knower on one task is also a 'one'-knower on all of the others, ditto for 'two'-knowers, 'three'-knowers and 'four'-knowers. The patterns of judgments across all of these tasks suggest that parallel individuation and the set-based quantification of natural

language underlie the numerical meanings subset-knowers construct for numeral words.

Also consistent with the claim of discontinuity, studies of nonverbal number representations in populations of humans who live in cultures with no count list (e.g. the Piraha; Gordon, 2004; Frank *et al.*, 2008), and populations of humans in numerate cultures with no access to a count list (e.g. homesigners, Spaepen *et al.*, 2011) show no evidence of any number representations other than the three CS1s.

In sum, the construction of the numeral list representation is a paradigm example of developmental discontinuity. How CS2 transcends CS1 is precisely characterized, and consistent with this analysis, CS2 is difficult to learn and not universal among humans.

10. Greater Expressive Power?

The above analysis makes precise the senses in which the verbal numeral list (CS2) is qualitatively different from those manifest representations with numerical content that precede it: it has a totally different format (verbal numerals embedded in a count routine), none of the CS1s with numerical content can express, even implicitly, an exact cardinal value over 4. But is the argument that the concepts for specific integers are new *primitives*, undefinable in terms of preexisting concepts using the combinatorial resources available to the child, actually correct? This argument, if correct, establishes the claim that acquiring the verbal count list representation of integers increases expressive power. As I comment in *TOOC*, this is on its face an odd conclusion. Integers are definable, after all, in terms of many different possible sets of primitives (e.g. 1 and the successor function, or the principle that 1-1 correspondence guarantees numerical equivalence plus the resources of quantificational logic).

Key assumes that the logical combination underlies the transition from CS1 (core cognition of number) to CS2 (representations of verbal numerals that implicitly express the successor function). This is only possible if the capacity to represent integers is innate (e.g. if there is an innate representation of 1 and the *successor function*), or if integers are definable, by logical construction, from *manifest* innate primitives using *manifest* logical processes of conceptual combination. I agree that whether learning integers increases expressive power simply is this question. Without a full characterization of the manifest combinatorial (logical) apparatus available to the child at the time the integers are constructed one cannot definitively answer the question of whether the child *could in principle* construct integer representations from innate resources, quite apart from the question of whether this is how the child *does* arrive at integer representations. But one can explore how the child actually *does* do so, and, in the remaining pages of this article, I explain why I believe the process is *not* one of logical construction.

It's true that humans must ultimately be able to formulate concepts of integers using the explicit machinery of logic, enriched by whatever numerical concepts are

necessary as well (what is actual is possible). But it is only after very long historical, and ontogenetic, developmental processes that the construction of integers in terms of logic or Peano's axioms is made. We simply do not know whether part of this process involved bootstrapping new logical representations as well as new numerical primitives.

11. A Logical Construction of the Cardinal Principle

Piantadosi *et al.* (2012) provided a proof of concept demonstration that children could, in principle, construct a count list representation of the integers (at least up to 'ten') by conceptual combination alone, given the full general resources of logic (in the form of lambda calculus, including the capacity for recursion, as well as specific logical and set operators such as *if/then* and set comparison functions), knowledge of the structure of the count list (its order), four numerical primitives: concepts of *singleton*, *doubleton*, *tripleton*, and *quadrupleton* (i.e. already manifest concepts of *one*, *two*, *three*, and *four*), as well as other logical and set operators (*if/then*; *set-difference*). This is a proof of concept in the sense of learning by a computer that has these resources. Piantadosi *et al.* appeal to the literature on learning to count in support of the claim that these numerical concepts and a representation of the count list are manifest at the time of the induction of the counting principles, but they merely assume—without evidence—that full general resources of lambda calculus and logic are available for the generation of hypotheses about what 'one', 'two', 'three', 'four', 'five' . . . through 'ten' mean. They assume that children learn the meanings of the words 'one' through 'ten' from hearing words in cardinal contexts, through Bayesian enumerative induction. Thus, their model satisfies Fodor's Premises 1 and 2.

The model receives input in the form of sets with 1 to 10 items paired with the appropriate verbal numeral. It learns a function, in the language of lambda calculus, that allows it to answer the question 'how many individuals?' with the correct numeral. The model's input reflects the relative frequency of verbal numerals in parental speech to children (i.e. 'one' is vastly more frequent than 'two', and so on.) Learning is constrained by limiting the combinatorial primitives that articulate hypotheses to be evaluated to those detailed above, by a preference for simpler hypotheses (i.e. shorter expressions in lambda calculus), and by a parameter that assigns a cost for recursion. After considering over 11,000 (!) different hypotheses composed from these primitives, the model learns to assign the words 'one' through 'four' to the concepts *singleton*, *doubleton*, *tripleton*, and *quadrupleton*, and also (independently) learns a recursive cardinal principle knower function that assigns the numerals 'one' through 'ten' to sets of one through ten individuals. The recursive function tests whether the set in question (S) is a singleton, and if so, answers 'one'. If not, it removes an element from S, using the set-difference primitive, and uses the recursive function to apply the same singleton probe on the resultant set. If the answer is yes, it then applies the function next (defined for the count list) and

outputs the next word in the list as the answer to the question, if not, it recursively repeats this step.

The model matches, qualitatively, several details of children's learning to count: children go through 'one'-, 'two'- 'three'- and 'four'-knower stages, in that order, and depending upon the cost assigned to recursion, learn the CP-knower function after becoming 'three'-knowers or 'four'-knowers. Before the model learns the recursive CP-function, it has no way of knowing what numeral to apply to sets greater than 4, and in this sense Piantadosi *et al.* claim a discontinuity in the model's knowledge of number word meanings. Thus, they claim for this model that it puts bootstrapping on a firm computational basis, as well as focusing on the logical resources actually needed for bootstrapping to succeed.

Piantadosi *et al.* assert that combination is the source of novelty. Therefore, in the current discourse, they are denying a genuine discontinuity. There is no change in expressive power—the manifest primitives (both numerical and logical) clearly can, in combination, express the cardinal meanings of 'one' through 'ten'. I will show why this model does not implement Quinean bootstrapping after I've discussed Quinean bootstrapping (see Rips, Bloomfield, and Asimuth, 2013, for an illuminating discussion). Here I simply want to acknowledge that, of course, depending upon the manifest concepts (both numerical and logical) actually available to the child, it certainly could be possible to learn the meanings of verbal numerals by constructing them from antecedently available concepts through logical combination.

The question that concerns me is how representations of integers *actually* arise in development. In what follows, I sketch a very different picture, one that does not rely on conceptual combination alone, and provide reasons to believe that this is the correct picture. My goal is to provide reasons to doubt that hypothesis formation by logical combination from primitives is the *only* source of new concepts.

12. Quinean Bootstrapping

In Quinean bootstrapping episodes, mental symbols are established that correspond to newly coined or newly learned explicit symbols. The latter are initially placeholders, getting whatever meaning they have from their interrelations with other explicit symbols. As is true of all word learning, newly learned symbols must necessarily be initially interpreted in terms of concepts already available. But at the onset of a bootstrapping episode, these interpretations are only partial—the learner does not yet have any manifest concepts in terms of which he or she can formulate the concepts the symbols will come to express.

The bootstrapping process involves aligning the placeholder structure with the structure of existent systems of concepts that are manifest in similar contexts. Both structures provide constraints, some only implicit and instantiated in the computations defined over the representations. These constraints are respected as much as possible in the course of the modeling activities, which include analogy

construction. When the bootstrapping is under metaconceptual control, as is the case when it being carried out by adult scientists, the analogical processes are explicit, and the fruitfulness of the analogies are monitored, and other modeling processes are also deployed, such as limiting case analyses, and thought experiments. Inductive inference is also often involved in bootstrapping, constrained by the actual conceptual structures in the process of being aligned.

In the case of the construction of the numeral list representation of the integers, the memorized count list is the placeholder structure. Its initial meaning is exhausted by the relations among the external symbols: they are stably ordered and applied to a set of individuals one at a time. 'One, two, three, four...' initially has no more meaning for the child than 'a, b, c, d...', if 'a, b, c, d...' were to be recited while attending to individuals one at a time.

The details of the subset-knower period suggest that the resources of parallel individuation, enriched by the machinery of linguistic set-based quantification, provide numerical meanings for the first few numerals, independently of their role in the memorized count routine. Le Corre and I (2007) proposed that the meaning of the word 'one' is represented by a mental model of a set of a single individual {i}, along with a procedure that determines that the word 'one' can be applied to any set that can be put in 1-1 correspondence with this model. Similarly 'two' is mapped onto a long term memory model of a set of two individuals {j k}, along with a procedure that determines that the word 'two' can be applied to any set that can be put in 1-1 correspondence with this model. And so on for 'three' and 'four'. This proposal requires no mental machinery not shown to be in the repertoire of infants—parallel individuation plus the capacity to compare models on the basis of 1-1 correspondence. But those representations are enriched with the long-term memory models that have the conceptual role of assigning 'one', 'two', 'three', and 'four', to sets during the subset-knower stage of acquiring meanings for verbal numerals. We suggested that enriched parallel individuation might also underlie the set-based quantificational machinery in early number marking, making possible the singular/plural distinction, and in languages that have them, dual and trial markers. The work of the subset-knower period of numeral learning, which extends in English-learners between ages 2:0 and 3:6 or so, is the creation of the long term memory models and computations for applying them that constitute the meanings of the first numerals the child assigns numerical meaning to.

Once these meanings are in place, and the child has independently memorized the placeholder count list and the counting routine, the bootstrapping proceeds as follows: The child must register the identity between the singular, dual, trial, and quadral markers and the first four words in the count list. In the course of counting the child notes (at least implicitly) the suspicious coincidence that the numeral reached when counting a set of 'three' is also the word a 'three'-knower takes to represent the cardinal value of that set. This triggers trying to align these two independent structures. The critical analogy is between order on the list and order in a series of sets related by *additional individual*. This analogy supports the induction that any two successive numerals in the child's finite count list will refer

to sets such that the numeral farther in the list picks out a set that is 1 greater than that earlier in the list.

In my earliest writings I characterized the induction made by 4-year-olds as yielding the first representations of integers. Let me be clear, as *TOOC* is, when the child has built the count list representation of the first 10 or so verbal numerals, the child does not yet have general representation of integers. There are many further bootstrapping episodes along the way to a representation of integers, two of which are discussed in *TOOC*—about 6 months after becoming CP-knowers, children construct a mapping between the count list and analog magnitude representations, yielding a richer representation of the meanings of verbal numerals (Chapter 9). Shortly thereafter, children abstract an explicit concept *number*, and explicitly induce that there is no highest number (Hartnett and Gelman, 1998). And it is not until late in elementary school or even high school that children construct a mathematical understanding of division that allows them to reanalyze integers as subset of rational numbers (Chapter 9). All of these developments are along the way to richer and richer representations of integers. But without the construction of an integer list representation of a finite subset of integers, which provides children with new primitive concepts for specific integers beyond four (e.g. ‘seven’ representing *exactly* seven) as well as providing new representations of ‘one’ through ‘four’ (in terms of their place in a count list, rather than only in terms of enriched parallel individuation), these further bootstrapping episodes never get off the ground.

This proposal illustrates all of the components of bootstrapping processes: placeholder structures whose meaning is provided by relations among external symbols, partial interpretations in terms of available conceptual structures, modeling processes (in this case analogy), and an inductive leap.

The greater representational power of the numeral list than that of any of the systems of core cognition from which it is built derives in part from creating a new representational structure—a count list—a new conceptual role—counting, and just *using it*. Much of the developmental process involves no hypothesis testing. Just as when the child learns a new telephone number (memorizes an ordered list of digits) and learns to use it in a procedure (dial, press buttons) that results in a ring and connection to Daddy, here the child learns an ordered list and procedure for applying it to individuals as one touches them one at a time. This new structure comes to have numerical meaning through the alignment of aspects of its structure with aspects of the structure of manifest number representations. These, in turn, have been built from set-based quantification (which gives the child singular, dual, trial, and quadral markers, as well as other quantifiers), and the numerical content of parallel individuation (which is largely embodied in the computations carried out over sets represented in working memory models with one symbol for each individual in the set). The alignment of the count list with these manifest meanings is mediated, in part, by the common labels (the verbal numerals) in both structures. At the end of the bootstrapping episode, the child has created symbols that express information that previously existed only as constraints on computations. Numerical content does not come from nowhere, but the process does not consist

of defining 'seven' by conceptual combination of primitives available to infants. 'Seven' is genuinely a new primitive, the meaning of which is provided in part by its conceptual role in a new conceptual structure.

With this characterization in hand, one can see why the Piantadosi *et al.* (2012) model does not implement a Quinean bootstrapping process. There are three theoretically important differences between Quinean bootstrapping and a model that formulates hypotheses at random by explicit conceptual combination from 15 primitives, one numeral at a time, and then uses Bayesian induction to evaluate them. First, although, like Piantadosi *et al.*, I assume that children have representations with the content *singleton*, *doubleton*, *tripleton*, *quadrupulton*, before the children induces the cardinal principles, the numerical content of these representations is carried by enriched parallel individuation, and is merely implicit until the child constructs the relevant structures. The first explicit symbols are 'one', 'two', 'three' and 'four' and their meanings are not already existing primitives *singleton*, *doubleton*, *tripleton*, *quadrupulton*. That is, there is no innate *explicit* symbol *doubleton* that can figure in hypotheses composed using lambda calculus. Similarly, the representations that underlie the meaning of *seven*, after the cardinal principle induction, are largely implicit in the procedures of the count routine, not explicitly defined in a language of thought. Second, the meanings of numerals in the Piantadosi model are learned entirely independently from each other. That is, children could, in principle, compose the recursive definition of numerals first, without ever going through 'one'-, 'two'-, 'three'-, and 'four'-knower stages. Indeed, the only parameters that affect the timing of CP-transition are the cost of recursion and the encountering of larger numerals in cardinal contexts. In Piantadosi's model, knowing the meaning of 'one' (as *singleton*) plays no role in learning the meanings of other numerals or becoming a CP-knower. In Quinean bootstrapping, the structure created by interrelations of the newly learned words, plus their partial meanings from initial mappings to prelinguistic thought, play an essential, constitutive role in the learning process. Thirdly, and relatedly, the Quinean bootstrapping story takes seriously the question on the source of constraints on the learning process. It empirically motivates its claims of the exhaustive set of primitives with numerical content, (the three CS1s), and provides evidence for syntactic bootstrapping as an account for how the child breaks into the meanings of the first numerals. As Rips *et al.*, 2013, point out in their illuminating discussion of the Piantadosi model, this model does not provide an account for how the hypothesis space is conveniently limited to just the 15 numerically relevant primitives it randomly generates hypotheses from. The child has much other numerically relevant knowledge at the time of the CP induction. If that knowledge were included in the set of primitives, the hypothesis space created by random combination from the primitives would explode beyond the already entirely unrealistic 11,000 hypotheses considered and rejected by the model. If numerically irrelevant primitives are included (how does the child decide which primitives are relevant?), the problem would quickly become entirely intractable.

In sum, Quinean bootstrapping is very different from the Piantadosi logical combination model, but which model provides better insight into how children

actually learn how counting represents number? Two recent animal studies clarify the nature of bootstrapping, allowing us to see that it does not involve hypothesis testing over a huge space of possessed but unmanifest concepts, nor does it involve logical combination of primitives. These studies also increase the plausibility that that young children have the computational resources to engage in Quinean bootstrapping.

13. Animal Models

In *TOOC* I speculated that Quinean bootstrapping might well be a uniquely human (depending upon external explicit symbols as it does), and thus might provide part of the explanation for the uniquely human conceptual repertoire. Since then, two studies have convinced me that other animals have the capacity for Quinean bootstrapping.

13.1 Alex

The first study (Pepperberg and Carey, 2012) drew on explicit numerical representations created by Alex, an African grey parrot, who had been trained by Irene Pepperberg for over 30 years. He had a vocabulary of over 200 words, including object labels, color words, relational terms like 'same', and several other types of labels. Alex had been taught to produce the words 'three' and 'four' in response to 'how many x?' for sets of 3 and 4 respectively. During this initial training, Alex was also shown mixed sets of objects (e.g. 4 blue balls, 5 red balls, and 3 yellow balls), and asked, for example, 'what color three', responding 'yellow'. In other words, he was first taught to produce and comprehend 'three' and 'four' as symbols for cardinal values 3 and 4. After this training was in place, he was similarly taught to produce and comprehend 'two' and 'five' as symbols for cardinal values 2 and 5. And then 'one' and 'six' were added to his repertoire.

We do not know what non-linguistic numerical representations underlay these explicit numeral representations, because we do not know the sensitivity of Alex's analog magnitude representations or the set size limit of his parallel individual system. Analog magnitude representations themselves could have done so, or both parallel individuation and analog magnitudes could have been drawn upon. As he is a non-linguistic creature, he doesn't have the resources of set-based quantification that is part of the language acquisition device to draw upon. What the quantificational resources of non-linguistic thought are has not been studied, but Alex clearly had the capacity to selectively attend to small sets and evaluate whether any given set had a cardinal value of 'one' through 'six'.

After he had a firm understanding of the cardinal meanings of 'one' through 'six', Pepperberg taught him to label plastic tokens of Arabic numerals '1, 2, 3, 4, 5' and '6', with the words 'one' through 'six' respectively. Arabic numerals were never paired with sets of individuals. The only connection between Arabic numerals and

set sizes was through the common verbal numeral (e.g. 'two' for '2' and 'two' for a set of 2 individuals.) He quickly learned to produce and comprehend the verbal numeral labels for the Arabic numerals. Then with no further training, Pepperberg posed him the following question for each pair of Arabic digits between '1' and '6': 'Which color number bigger?'. He was to choose, for example, between a blue '3' and a red '5', the Arabic numeral tokens being the same size and the correct answer being 'red'. He succeeded at this task when first presented it; it required no further training. Not only had he not been given any positive evidence that '2' refers to a cardinal value, the only context in which he had answered questions about 'bigger' and 'smaller' previously was in with regards to physical size (i.e. 'which color bigger' of two objects that differed in size).

Please dwell on this finding. It must be that the common labels (e.g. 'two') had allowed him to connect a representation of the Arabic digits (e.g. '2') with the cardinal values (e.g. 2), and it must be that the intrinsic order in his nonverbal representations of cardinal values allowed him to say which Arabic numeral was bigger or smaller than which others. Although Alex had never been taught a count list (and had been taught the cardinal meanings of numerals in the order 'three/four', 'two/five' and finally 'one/six'), by the time we began our study Alex could produce and comprehend the words 'one' through 'six' as labeling both cardinal values of sets and Arabic digits, and could use the intrinsic order among set sizes to order his verbal numerals.

We were thus in a position to teach Alex to label Arabic numerals '7' and '8', 'seven' (pronounced by him 'sih-none' and 'eight' respectively.). This training took about a year, and during it he had no evidence that '7' or '8' were numerals. He was then taught that '6' is a smaller number than '7', which in turn is a smaller number than '8', by posing the 'which color number bigger/smaller' task, giving him feedback if he guessed wrong. This was the first evidence he had that '7' and '8' are numerals, as are '1' through '6'. It took only a few hours to train him to answer 'which color number bigger?' and 'which color number smaller?' for each of the pairs: '6/7', '6/8' and '7/8'. After he had reached criterion on this task he was probed which color number bigger and smaller for each pair of numerals '1, 2, 3, 4, 5, 6' with respect to '7' and '8', and succeeded at this task with no further training. Thus, at this point he knew that '7' and '8' are numerals, labeled 'sih-none' and 'eight' respectively, and he knew that '8' is a bigger number than '1' through '7' and '7' is a bigger number than '1' through '6'. Importantly, he had never been given any information about which cardinal values 'sih-none/7' and 'eight/8' referred to.

The question of this study was whether he would make the (wildly unwarranted) induction that 'sih-none/7' expresses cardinal value 7 and 'eight/8' expresses cardinal value 8. He did. The very first time he was asked to label a set of seven object 'how many treats?' he answered 'sih-none' and the first time he was asked to label a set of eight objects 'how many treats?' he said 'sih-none' and immediately self corrected to 'eight'. Over a two-week period he was asked to label sets of different sizes (these questions were probed by many different experimenters, only

a few questions a day, intermixed with many other questions currently under study, concerning visual illusions and many other things. He performed better than chance producing both 'sih-none' and 'eight' ($p < .01$ in each case). He was also given comprehension trials, (e.g. 'what color seven?' and 'what color eight?', each question probing three sets or either 6, 7, 8, 9, or 10 colored blocks), and got 11 of 12 correct ($p < .01$). Thus, Alex had inferred the cardinal meanings of 'eight' and 'seven/sih-none' from knowledge of the cardinal meanings of 'one' through 'six' and from the fact that six is a smaller number than seven and seven is a smaller number than eight.

The Piantadosi model could not possibly apply here. This learning episode did not involve hypothesis confirmation. Alex never got any feedback as to whether his answers were correct. Nor was he ever given the pairings between 'seven (sih-none)' and sets of 7 and 'eight' and sets of eight that constitute the data for the Piantadosi model. Alex *must* have made an inductive inference based on the meanings of numerals he already had constructed. Given that his knowledge of the use of numerals was exhausted by just a few procedures they entered into (answering questions about set size and numerical order, labeling cardinal values of sets and labeling Arabic numerals), and by the mappings he had already made between representations of sets, verbal and Arabic numerals, his induction was subject to strong constraints. He clearly had not searched through a vast unconstrained hypothesis space specified by logical combination of all 250 or so concepts he had that were lexicalized (or even a larger set of conceptual primitives he may possess). As mentioned, this induction was wildly unwarranted; what he had been taught was consistent with '7' referring to any set size greater than '6' and with '8' referring to any set size greater than whatever '7' refers to. But in his 30 years of working with numerals, they had been introduced as related by +1 ('three' versus 'four', then 'two' and 'five', and then 'one' and 'six' added). His induction was not mathematically or logically warranted, but it was sensible, given his actual experience with numerals. So too is the child's.

Piantadosi *et al.* might reply that Alex may have made the leap to CP knower, having engaged in the same conceptual combination process as hypothesized by their model that children do, during the period of learning where he was taught 'one' through 'six'. In that case, the induction he made here was that 'seven' and 'eight' were the next two numerals, in that order, in the relevant list after 'six'. This is also not possible, because Alex lacked an essential set of primitive functions for the Piantadosi model during this earlier period: namely, he did not have a count list. He was never taught a list, *per se*, nor ever taught to count. Thus, he could not form any generalizations carried by the function Next applied to a count list. He wasn't even taught the numerals in numerical order (remember he learned first 'three' and 'four', then 'two' and 'five' and finally 'one' and 'six'. It's true he explicitly knew his numerals were ordered, but that order had to be derived from by the intrinsic order of cardinal values that were expressed by numerals and could not have been part of the source of the mapping between numerals and cardinal values. That order was not carried by a count routine and a memorized ordered

list. Further insight into the process of learning Alex was more likely engaged in is provided by a recent study of rhesus macaques.

13.2 Rhesus Macaques

Livingstone *et al.* (2009) taught four juvenile male rhesus macaques (1-year-old at beginning of training), to choose the larger of two dot arrays, or to choose a symbol that came later in an arbitrary list. The dot arrays varied between 1 and 21 dots, and the arbitrary list of symbols was '1, 2, 3, 4, 5, 6, 7, 8, 9, X, Y, W, C, H, U, T, F, K, L, N, R'. The monkeys were trained on the dot arrays and on the symbol list on alternate days. Training in both cases involved giving the monkey a choice between two stimuli (e.g. 2 dots and 7 dots, or '2' and '7') on a touch screen. When the monkey touched one of the arrays, he was rewarded with the number of pulses of juice or water that corresponded to his choice. Thus, he was always rewarded, but got bigger rewards for picking the larger dot array or the symbol later in the list. The monkeys learned to pick the stimulus that led to the larger reward with both stimuli sets, and were extremely accurate with both types of stimuli, making errors only for closely adjacent values.

There were two extremely interesting results that emerged from this study. First, with no training, the first time monkeys were given a choice between dot arrays and symbols (e.g. 4 dots and '7'), they chose the stimulus that would lead to the larger reward. That is, they had automatically integrated the two predictors of quantity of liquid—dot arrays and discrete symbols ordered in a list. Clearly this integration had to be mediated by the fact that the dot array and discrete list tasks established a common context (same testing chamber, same dependent measure of touching one of two stimuli on a screen), and the outcomes predicted were from the same scale of quantities of liquid. Still, they had integrated them. This is the structural alignment process drawn upon in bootstrapping.

Second, when making a choice between dot arrays, the noise in choices among large sets (e.g. 19 versus 21) was greater than that between smaller sets (e.g. 9 versus 11 or 3 versus 5). In fact, the choices showed scalar variability, the marker of analog magnitude values (see above). But errors in when choosing values on the ordered list of discrete symbols did not increase as the list got longer. Livingstone *et al.* interpreted this difference as showing that the mapping from dot arrays to liquid quantity showed scalar variability, whereas the mapping from the list to hedonic value was linear. A more likely interpretation is that the mapping, during learning, reflected recognizing the relevance of each type of order (order among set sizes in analog magnitude representations of number of dots, and linear order in an arbitrary list) and inducing a rule that one should pick the stimulus later in each ordering. It's analog magnitude representations of dots that showed scalar variability, and the representations of the linear order in the list that did not. It's true that some mapping between each ordering and quantity of liquid was constructed in the process, because the two orderings were integrated. But if choosing between predicted quantities of liquid underlay each choice, both tasks should have shown

scalar variability, since quantity of liquid is represented with an analog magnitude value. I suggest that the structure of an ordered list of symbols is a linear order, supported by the discriminability of each symbol from each other, and this order directly determined choice once the task was learned. This structure, after being constructed, was alignable with the intrinsic order of representations of quantity of liquid, and then with the other predictor of quantity of liquid (dot arrays). This is structurally the same as the alignment between an ordered list and analog magnitude representations of number achieved some 6 months after children have become cardinal principle knowers.

Livingstone's rhesus macaques did not induce the cardinal meaning of a new symbol from its place in a count list, but nonetheless they exhibited several components of the extended bootstrapping process that supports children's (and Alex's) doing so. They did build a representation of an ordered list (21 elements long!) and aligned it with a representation that was itself intrinsically ordered. Also, they automatically aligned two different ordered representations (the list, the dot arrays) which were separately aligned to quantity of liquid. Clearly, finding the structural correspondence between an ordered list and increasing magnitude (whether that magnitude is number or a continuous variable like quantity of liquid) is a natural computation, at least for primates.

14. Conclusions Concerning the Nature of Quinean Bootstrapping

As the historical examples discussed in *TOOC* make clear, bootstrapping episodes are often under metaconceptual control; the scientist is consciously engaged in exploring mappings between mathematical structures and physical/biological/psychological phenomena. But as the above examples from animal learning make clear, metaconceptually explicit hypothesis testing and modeling procedures are not necessary.

I now turn to the questions of whether the representations achievable by bootstrapping should be thought of as a preexisting hypothesis space, and whether the mechanism is exhausted by hypothesis formation and confirmation and by logical combination of existing primitives.

First, prior to the bootstrapping processes, neither children, nor Alex, nor rhesus macaques have any representations for exact cardinal values outside of the range of parallel individuation. A representations of 341,468, or of 10, does not exist in some preexisting hypothesis space ready to become manifest. Some of the learning processes involved in this extended episode are certainly not hypothesis testing (e.g. memorizing the ordered list of numerals), and some are subpersonal (as Shea [2011] put it, 'not explainable by content;' see also Stevens' [2012] proposal that introjection involves subpersonal processes). That is, the connection of the 'three' in count list with the 'three' of enriched parallel individuation is most probably mediated simply by the shared label and associative machinery, just as Alex's aligning of his representations of verbal numerals, set sizes, and Arabic numerals is

based first on common labels, which then supports ordering them according to the intrinsic order among cardinal values within AM and parallel individuation systems of representations. Alex never got any feedback regarding the pairing of 'seven' and 'eight' with cardinal values, so no hypothesis confirmation or Bayesian enumerative induction was involved. Similarly, the rhesus' aligning of an ordered list of 21 discrete symbols with set sizes from 1 to 21 depends upon shared associations with quantities of liquid. Such alignment processes are not processes of logical combination (although logical combination is involved in building the placeholder structures). I conclude that Quinean bootstrapping yields new primitives in this case, representations of integers embedded in a count list, and is a learning mechanism that does not conform to Premises 1 and 2 of Fodor's argument.

15. Rey's (and Others') Critiques of Quinean Bootstrapping

At the end of his commentary, Rey comments 'any process of learning can only take place against a background of an innate repertoire of concepts'. I cannot imagine he would think I would deny that, since I explicitly say the same thing throughout *TOOC*, and virtually everybody on all sides of these debates agrees, although not everyone likes the word 'innate'. For example, the empiricists' proposal for the innate repertoire that they knew was needed was that it consists of 'sensory ideas'. The CS1s are, by hypothesis, the only innate representations with numerical content, and their characterization includes specific proposals concerning the nature of the symbols involved and computations supported. These representations are the sources of the inductions the child makes, as well as the source of the constraints on those inductions.

Rey denies Quinean Bootstrapping is a learning mechanism that can increase expressive power by creating new primitives not laying in wait. He also denies that Quinean bootstrapping actually creates new primitives not constructable by logical combination from others. Specific versions of his challenges include 1) analogy cannot create new representational resources, as analogies require alignable structures antecedently, 2) the induction the child makes requires an antecedent appreciation of the successor function, and 3) the bootstrapping proposal fails to confront Goodman's 'grue' problem, the problem of constraints on induction. As I hope is already clear, I believe all of these challenges to be off the mark.

With respect to the challenge that analogy requires already available representations to be aligned, I agree. Rey misses that the bootstrapping process is an extended one. The new representational resource is not created at the moment of the analogy and the induction alone. By the time of the induction of the counting principles, the child has indeed created the alignable structures needed for the limited induction he/she makes, just as Alex had. In the case of the child these structures are, by hypothesis, the count list and representations of the cardinal values of the numerals 'one' through 'four' supported by enriched parallel individuation. The whole process begins with the innate numerical resources (the CS1s described

above), the enrichment of parallel individuation during the subset-knower stage, and the creation of the meaningless placeholder structure. Of course one needs both structures to align them; the bootstrapping process accounts for the origin of each structure and shows what new arises from this alignment.

I don't agree with Rey's second critique, that to notice sets of two differ from sets of three by a single individual, one must already represent the successor function. All one must be able to do is subtract 2 from 3, and 1 from 2, computations that both parallel individuation and analog magnitude representations support. The successor function, in contrast, generates an infinite series of cardinal values, but the knowledge the child has is initially restricted to the relations among sets of one, two, three and four (because of the set size limit on parallel individuation and the sensitivity of analog magnitude representations being limited to 3:4 or 4:5 among young preschoolers.). But of course, without the capacity to subtract 2 individuals from a set of 3 individuals and 1 individual from a set of 2 two individuals, yielding 1 individual in each case, the child could not make the induction concerning how his or her short count list works. I do not deny this capacity must be in place for the induction; rather I present *evidence* that it is, including *how* it is (within the system of enriched parallel individuation in the case of children's learning to count), and *evidence* that precisely that induction separates subset-knowers from cardinal principle-knowers. Again, one must consider the format and computational roles of the actual representations involved. The induction the child most probably makes is that when you add an individual to a set for which you would reach numeral N when applying the count routine, if you count the resulting set, you will reach the next word on the count list. This is not yet the successor function, and certainly doesn't presuppose the successor function.

Turning to the heart of Rey's criticism: that I failed to take on the psychological version of Goodman's new riddle of induction. Goodman's concern was providing a valid warrant for inferring 'all emeralds are green' in preference to 'all emeralds are grue'. Neither the induction Alex made nor that of children is warranted. As Rey makes clear, the psychological question Rey and I are concerned with is explaining why we *don't* entertain the latter generalization when considering the color of emeralds, or why children don't entertain the hypothesis that 'five' is a proper name for a set or the last number word in a mod 6 modular arithmetic count list. This article has been an extended response to the critique that the bootstrapping story fails for failing to answer this question. Human inductive inference is profligate; so too, apparently, is parrot inductive inference. Accounting for the constraints on induction is everybody's problem. Where Rey goes wrong derives from his view of possessed concepts as a vast hypothesis space, laying in wait to become manifest. If this were right (think Piantadosi *et al.*), the issue of constraints on induction would indeed be trenchant. As I have argued, I think this the wrong way to think about concept possession, as well as the wrong way to think about concept acquisition. As I have already said, one can always explore the possible outputs of proposed learning mechanisms, thus exploring concept possession in the unproblematic sense of potential final states. And as I have also already said, any actual learning mechanism

imposes constraints on what can be learned. Thus, part of the project of exploring an actual learning mechanism is studying what constraints are imposed by it, including constraints on induction when induction is involved (which is not always). Positive characterizations of learning mechanisms always specify the constraints under which they operate.

One understands the constraints on the inductions made by 3-year-olds and by Alex by attending to the extremely limited contexts in which inductions are usually drawn (think Alex and the rhesus macaques, as opposed to the model of Piantadosi *et al.*, selecting among over 11,000 hypotheses consistent with the data it has received, where that large hypothesis space has been artificially constrained). The induction made during the hypothesized bootstrapping episode during childhood, and by Alex, is constrained by the structures being aligned, and their very local conceptual roles. The scientific work involved in understanding episodes of Quinean bootstrapping is characterizing those structures, showing how they arise, and showing what new is achieved by aligning them.

16. Final Conclusions

I have argued, contrary to Rey's critique, that I have not confused an epistemic question with a logical/semantic one, and that increases of expressive power in the logical/semantic sense are commonplace, due to a variety of learning mechanisms that result in new conceptual primitives. In cases where this is easy for the learner, the concepts to be acquired are constrained by already existing conceptual roles (such as a conceptual role for a representation of the North Star, to guide migration in celestial navigators, of the general conceptual roles for representations of object kinds in general, and artifact kinds and agent kinds, in particular). In cases where this is hard for the learner, the learner must create new conceptual roles (part of the bootstrapping process), which then constrain further learning in a way analogous to the easy cases. In neither case does the learner search among a vast space of possessed but unmanifest concepts, nor does conceptual novelty arise only by combining primitives using the combinatorial machinery of logic.

I embrace Rey's ecumenical proposal that concept manifestation is a worthy project for psychological study, although I reject his picture of concept manifestation as some mysterious process through which already existing, possessed but unaccessible, concepts become available for thought. Concepts we can think with are the output of actual learning mechanisms that in turn provide actual constraints on learning, including constraints on induction when induction is part of the learning process. The characterization of those learning mechanisms is where the explanation of the human conceptual repertoire will play out.

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